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COMPUTER RESEARCH OF THE MATHEMATICAL MODEL OF ETHNIC GROUP DEVELOPMENT

This article highlights the main factors - subsystems that affect the development processes of ethnic groups. A mathematical model of ethnic groups is constructed and a computer study of this system is carried out.

A qualitative study of the dynamic system of ethnic groups is carried out for the case when political differentiation and the degree of adaptation is constant. Relations are written out characterizing the number of equilibrium states, their type, the number of singular points at infinity, and phase portraits are presented using application programs.

This computer study on a given set of characteristics will allow you to simulate the occurrence of problem situations and make changes in advance.

Key words*: *computer research, mathematical model, ethnos, ethnic groups, passionarity, passionarity.

With the development of digital technology in the humanities, tools and approaches of the natural sciences have become increasingly used. Today, mathematical methods are widely used in sociology, economics, medicine, politics, etc.

For a mathematical description of ethnic groups in Kazakhstan, Talcott Parson's systematic approach will be used and based on this approach we will compose differential equations describing the dynamics of changes in the phase variables of the system under consideration, which is equivalently constructed to the A.K. dynamic system Gutsa [1] and A.A. Laptev [2]. The system consists of constituent elements that are interconnected and interdependent. Elements have integral properties, they strive to maintain the integrity of the system. Each change in the position of one part inevitably leads to a change in other parts [3].

At the Parsons system level, we distinguish four subsystems of ethnic systems of Kazakhstan, each of which performs one of four main functions: economic, designed to ensure the adaptation of the system to the environment, political, the purpose of which is to achieve the goal, ethnic groups (a single team that obeys a certain accepted normative order), which ensures internal unity, and the governing institute of ethnic systems of Kazakhstan (Assembly of the Peoples of Kazakhstan (APK)), which is responsible for the legitimization of the normative order and Saving the state of unity [4].

The main control parameter is the passion voltage P. The components of the dynamic system of ethnic groups are described as follows: L(t)- this is a function that describes the political system; E(t)- economic system, T(t) - function describing the development of ethnic groups; H(t)- function describing the dynamics of the APK Institute.

Thus, the model of ethnic groups in Kazakhstan is a system of four differential equations:

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$$\begin{cases} \dfrac{dL}{dt} = k_{LL}(e^{\delta P-\delta_1}-1)\cdot L + k_{LE}e^{-\mu E+\mu_1}\cdot E + k_{LT}(P-P_1)(T+H)\cdot L \\ \dfrac{dE}{dt} = k_{EE}(e^{\delta P-\delta_1}-1)\cdot E + k_{EL}e^{-\gamma E+\gamma_1}\cdot L + k_{ET}(P-P_2)(T+H)\cdot E \\ \dfrac{dT}{dt} = k_{TL}(L^2-E^2)k_1\cdot T - k_{TH}\cdot H^2 \\ \dfrac{dH}{dt} = k_{HL}\cdot L^2 - k_2\cdot H - k_{HT}\cdot T^2 \\ L|_{t=0}=L_0,\ E|_{t=0}=E_0,\ T|_{t=0}=T_0,\ H|_{t=0}=H_0, \end{cases} \quad (1)$$

Let's consider the case when L-political differentiation and E - degree of adaptation are constant, i.e. fixed at some level and do not change over time. Then, in this case, we can assume that these variables are equal to some constants.

$$\begin{cases} \dfrac{dT}{dt} = k_{TL}(L^2+E^2) - k_1\cdot T - k_{TH}\cdot H^2 \\ \dfrac{dH}{dt} = k_{HL}\cdot L^2 - k_2\cdot H - k_{HT}\cdot T^2 \\ T|_{t=0}=T_0,\ H|_{t=0}=H_0 \end{cases} \quad (2)$$

To simplify the study, assume that the following coefficients are equal:

- coefficients that characterize the share of political systems that affect the change of ethnic groups and the institution of the Assembly of people of Kazakhstan (APK), so $k_{TL} = k_{HL}$;

- intensity of losses of ethnic groups and the Institute of the Assembly of people of Kazakhstan, so $k_1 = k_2$;

- coefficients of mutual influence of ethnic groups and the Institute of the Assembly of people of Kazakhstan, so $k_{TH} = k_{HT}$.

Let's introduce new designations: $s_1 = k_{TL}(L^2 + E^2)$, $s_2 = k_{HL} \cdot L^2$, $k = k_1 = k_2$, $l = k_{KD} = k_{DK}$ and we assume that s_1, s_2, k, l are positive coefficients and k depends linearly on the positive parameter P. And note that $s_1 \leqslant s_2$. Also, we denote the variables T and H on x and y, respectively, and we do not impose any restrictions on the initial data T_0, H_0.

Then we get a system of equations:

$$\begin{cases} \dfrac{dx}{dt} = s_1 - k\cdot x - l\cdot y^2, \\ \dfrac{dy}{dt} = s_2 - k\cdot x - l\cdot y^2, \\ x|_{t=0}=x_0,\ y|_{t=0}=y_0, \end{cases} \quad (3)$$

Let's follow the change in the qualitative picture of the solution of the system (3) depending on the parameter P. To do this, we study the possible equilibrium states and directions along which trajectories can tend to them. We find the number of equilibrium states. To do this, we find the singular points equating the right-hand sides of the system of equations (3) to zero.

To study the equilibrium state, we consider the right-hand sides of the system of equations (3):

$$R(x_0, y_0) = s_1 - k \cdot x_0 - l \cdot y_0^2,$$

$$S(x_0, y_0) = s_2 - k \cdot y_0 - l \cdot x_0^2.$$

(x_0, y_0) - the nature of the equilibrium state is determined by the sign of the following three quantities:

$$\Delta = \Delta(x_0, y_0) = \begin{vmatrix} R'_x(x_0, y_0) & R'_y(x_0, y_0) \\ S'_x(x_0, y_0) & S'_y(x_0, y_0) \end{vmatrix} = k^2 - 4l^2 x_0 y_0,$$

$$\sigma = \sigma(x_0, y_0) = R'_x(x_0, y_0) + S'_y(x_0, y_0) = -2k_,$$

$$\sigma^2 - 4\Delta = 16l^2 x_0 y_0.$$

Then for functions Δ:

• in the I and III quarters the sign of the functions Δ is positive, for $k^2 - 4l^2 > 0$, so $k < -2l$ и $k > 2l$; otherwise, so for $2l < k < -2l$ the sign of the functions Δ is negative;

• in the II and IV quarters the sign of the functions Δ is positive, for any value of *k* and *l*.

And the signs of the functions $\sigma^2 - 4\,\Delta$ (for any value of *k* and *l*):

• in the I and III quarters are positive;

• and in the II and IV quarters are negative.

Thus, we obtain the following types of equilibrium:

1. When $2l < k < -2l$, saddle is in the I and III quarters, because $\Delta < 0$

2. When $k < -2l$ and $k > 2l$, node is in the I and III quarters, because $\Delta > 0$, $\sigma^2 - 4\Delta > 0$

3. Focus is in the II and IV quarters, because $\Delta > 0$, $\sigma^2 - 4\Delta < 0$.

Given initial conditions on the function $L\big|_{t=0} = L_0, E\big|_{t=0} = E_0, T\big|_{t=0} = T_0, H\big|_{t=0} = H_0$, implemented a computer model of ethnic groups. Let's follow the change of the phase portraits of the system (3), depending on the parameter *P*. The parameter *P* is present as a multiplier in the coefficients $k_1 = k_{TT}e^{-\eta E + \eta_1} \cdot P$ and $k_2 = k_{HH}e^{-\theta E + \theta_1} \cdot P$, but is not explicitly included in the number of coefficients of the system (3 Therefore, we introduce other new coefficient designations: $k_3 = k_{TT}e^{-\eta E + \eta_1}$, $k_4 = k_{HH}e^{-\theta E + \theta_1}$ and get a system of equations:

$$\begin{cases} \dfrac{dx}{dt} = s_1 - k_3 \cdot P \cdot x - l \cdot y^2, \\ \dfrac{dy}{dt} = s_2 - k_4 \cdot P \cdot y - l \cdot x^2, \\ x\big|_{t=0} = x_0,\ y\big|_{t=0} = y_0, \end{cases} \tag{4}$$

A computer study was used to trace the dynamics of the relative position of two parabolas, as well as curves, which allow us to determine the type of equilibrium state: $\Delta(x, y) = 0$, $\sigma^2(x, y) - 4\Delta(x, y) = 0$, где

$$\Delta(x, y) = k_3 \cdot k_4 \cdot P^2 - 4l^2xy$$
$$\sigma(x, y) = -k_3 \cdot P - k_4 \cdot P,$$
$$\sigma^2(x, y) - 4\Delta(x, y) = P^2(k_3 - k_4)^2 + 4l^2xy.$$

The computer study made it possible to effectively consider the change in the solution pattern with changes in the parameter P of values from a certain numerical segment and confirm the results of determining the types of equilibrium states obtained analytically.

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ЭТНИКАЛЫҚ ТОП ДАМУЫНЫҢ МАТЕМАТИКАЛЫҚ МОДЕЛІН КОМПЬЮТЕРЛІК ЗЕРТТЕУ

Бұл мақалада этникалық топтардың даму процестеріне әсер ететін негізгі факторлар – ішкі жүйелер көрсетілген. Этникалық топтардың математикалық моделі құрылып, осы жүйенің компьютерлік зерттеуі жүргізілді.

Саяси дифференциация мен бейімделу дәрежесі тұрақты болған жағдайдағы этникалық топтардың динамикалық жүйесіне сапалы зерттеу жүргізілді. Тепе-теңдік күйлерінің санын, олардың түрін, шексіздіктегі ерекше нүктелердің санын сипаттайтын қатынастар көрсетілген және қолданбалы бағдарламалар көмегімен фазалық портреттер келтірілген.

Берілген сипаттамалар жиынтығы бойынша жүргізілген осы компьютерлік зерттеу проблемалық жағдайлардың туындауын алдын алуға және уақытында өзгерістер енгізуге мүмкіндік береді.

Түйін сөздер*: *компьютерлік зерттеу, математикалық модель, этнос, этникалық топтар, пассионарлық, пассионарлық кернеу.

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КОМПЬЮТЕРНОЕ ИССЛЕДОВАНИЕ МАТЕМАТИЧЕСКОЙ МОДЕЛИ РАЗВИТИЯ ЭТНИЧЕСКОЙ ГРУППЫ

В данной статье выделены основные факторы – подсистемы, влияющие на процессы развития этнических групп. Построена математическая модель этнических групп и проведено компьютерное исследование этой системы.

Проведено качественное исследование динамической системы этнических групп для случая, когда политическая дифференциация и степень адаптации постоянно. Выписаны соотношения, характеризующие число состояний равновесия, их тип, количество особых точек на бесконечности, и приведены фазовые портреты с помощью прикладных программ.

Данное компьютерное исследование по заданному набору характеристик позволит имитировать возникновение проблемных ситуаций и заблаговременно вносить изменения.

Ключевые слова*: *компьютерное исследование, математическая модель, этнос, этнические группы, пассионарность, пассионарное напряжение.