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A CRITERION FOR UNIQUE SOLVABILITY OF A MULTIPOINT BOUNDARY VALUE PROBLEM FOR SYSTEMS OF INTEGRO-DIFFERENTIAL EQUATIONS WITH INVOLUTION

On the interval [0, T], a multipoint boundary value problem is considered for the systems of integro-differential equations with an involution transformation, when the kernel of the integral term is degenerate. Using the involution property, the problem is reduced to a multipoint boundary value problem for the systems of integro-differential equations with a degenerate kernel. It is shown that by introducing new parameters, changing variables and using the degeneracy of the kernel, the unique solvability of the original problem can be reduced to the inversibility of the resulting matrix.

***Key words**: boundary value problem, system of integro-differential equations, involution, involutive transformations, parameterization method.*

Introduction. It is known that many processes with aftereffect are described by integro-differential equations, for example, mechanics of a deformable solid, the theory of behavior of polymer materials, etc. The problem of e solvability of integro-differential equations is considered in the works of many authors [1-4].

Although numerous works have been devoted to the study of the solvability of boundary value problems for integro-differential equations, many questions of the qualitative theory still remain unsolved. On the basis of research in this field in 1989, Professor D. Dzhumabaev founded and proposed a method of parametrization [5], which was later applied to the study of boundary value problems for systems of integro-differential equations [6]. Various boundary value problems were considered using the parametrization method in [7-11].

In this paper we consider the boundary value problem on the interval .

$$\frac{dx(\mathrm{t})}{dt} + A\frac{dx(\alpha(t))}{dt} = \sum_{k=1}^{N}\int_0^T \phi_k(t)\psi_k(s)x(s)ds + f(t), \quad \mathrm{t}\in[0,T], \tag{1}$$

$$\sum_{i=0}^{m} B_i x(\theta_i) = d \quad , d\in R^n, \tag{2}$$

$$0=\theta_0<\theta_1<\ldots<\theta_{m-1}<\theta_m=T,$$

where matrices $\phi_k(t), \psi_k(s)$ and n - dimensional vector function $f(t)$ are continuous on [0, T]. Here $\alpha(t)$ is the homeomorphism $\alpha:[0,T]\to[0,T]$ changing orientation such that $\alpha^2(t)=\alpha(\alpha(\mathrm{t}))=\mathrm{t}$. Such homeomorphism is called an involution transformation. On the interval [0, T], as such a transformation, we can consider the homeomorphism $\alpha(t)=\mathrm{T}-t$.

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The problem of solvability of various differential equations with involution is discussed in the monograph of D. Przeworska-Rolewicz [12] and by J. Wiener [13]. W.B. Fite [14] described the properties of solutions to the equation with reflection $\alpha(t)=A-t$. Further, the properties of this homeomorphism were studied in the works of G.S. Litvinchuk [15], N.K. Karapetyants and S.G. Samko [16]. In [17, 18], spectral problems for differential equations and operators of the first and second orders with involution were studied. A series of works of Alberto Cabada and F. Tojo is devoted to the development of the theory of Green's function of one-dimensional differential equations with involution [19, 20].

Let us consider the values of equation (1) in the point $t=\alpha(t)$.

$$\frac{dx(\alpha(t))}{dt}+A\frac{dx(t)}{dt}=\sum_{k=1}^{N}\int_{0}^{T}\phi_k(\alpha(t))\psi_k(s)x(s)ds+f(\alpha(t)),t\in[0,T],$$

From the system

$$\begin{cases}\dfrac{dx(t)}{dt}+A\dfrac{dx(\alpha(t))}{dt}=\displaystyle\sum_{k=1}^{N}\int_{0}^{T}\phi_k(t)\psi_k(s)x(s)ds+f(t),\\ \dfrac{dx(\alpha(t))}{dt}+A\dfrac{dx(t)}{dt}=\displaystyle\sum_{k=1}^{N}\int_{0}^{T}\phi_k(\alpha(t))\psi_k(s)x(s)ds+f(\alpha(t)),\end{cases}$$

Multiplying the second equation by the matrix $-A$ on the left side, and adding the equations, we get

$$\left[I-A^2\right]\frac{dx(t)}{dt}=\sum_{k=1}^{N}\int_{0}^{T}\left[\phi_k(t)-A\cdot\phi_k(\alpha(t))\right]\psi_k(s)x(s)ds+\left[f(t)-Af(\alpha(t))\right].$$

Suppose that the matrix $\left[I-A^2\right]$ is not degenerate, then the boundary value problem (1), (2) can be written as:

$$\frac{dx}{dt}=\sum_{k=1}^{N}\int_{0}^{T}\tilde{\phi}_k(t)\psi_k(s)x(s)ds+\tilde{f}(t),\quad t\in[0,T],\tag{3}$$

$$\sum_{i=0}^{m}B_i x(\theta_i)=d\ \ ,d\in R^n,\tag{4}$$

$$0=\theta_0<\theta_1<\ldots<\theta_{m-1}<\theta_m=T,$$

where $\tilde{\phi}_k(t)=\left[I-A^2\right]^{-1}\cdot\left[\phi_k(t)-A\cdot\phi_k(\alpha(t))\right]$, $\tilde{f}(t)=\left[I-A^2\right]^{-1}\cdot\left[f(t)-Af(\alpha(t))\right]$.

Research methodology and results. Let us take a natural number $l\in\mathbb{N}$ and make a partition with respect to it: $[0,T)=\bigcup_{r=1}^{m(l+1)}[t_{r-1},t_r)$, where $t_{i(l+1)+j}=t_{i(l+1)}+\dfrac{\theta_{i+1}-\theta_i}{l}$, $i=\overline{0,m-1},\ j=\overline{1,l+1}$.

Suppose that $x(t)$ is a solution of equation (1) and $x_r(t)$ is its narrowing in the r – th interval $[t_{r-1}, t_r)$, $r=\overline{1,m(l+1)}$, i.e. $x_r(t)=x(t)$, $t\in[t_{r-1},t_r)$, $r=\overline{1,m(l+1)}$. Then the system of functions $x[t]=\left(x_1(t),x_2(t)...x_{m(l+1)}(t)\right)$ belongs to the space $C([0,T],\Delta_{m(l+1)},R^{nm(l+1)})$, and its elements $x_r(t)$, $r=\overline{1,m(l+1)}$ are continuous in $[t_{r-1},t_r)$, $r=\overline{1,m(l+1)}$ иand have finite left-hand limits $\lim\limits_{t\to t_r-0} x_r(t)$ при $r=\overline{1,m(l+1)}$, with the norm

$$\|x[\cdot]\|_2=\max_{r=\overline{1,m(l+1)}}\sup_{t\in[t_{r-1},t_r)}\|x_r(t)\|$$

Then problem (3), (4) is reduced to the equivalent problem:

$$\frac{dx_r}{dt}=\sum_{j=1}^{m(l+1)}\sum_{k=1}^{N}\int_{t_{j-1}}^{t_j}\tilde{\phi}_k(t)\psi_k(s)x_j(s)ds+\tilde{f}(t),\quad t\in[t_{r-1},t_r),\quad r=\overline{1,m(l+1)} \tag{5}$$

$$\sum_{i=0}^{m-1}B_i x_{i(l+1)+1}(t_{i(l+1)})+\lim_{t\to T-0}x_{m(l+1)}(t)=d \tag{6}$$

$$\lim_{t\to t_s-0}x_s(t)=x_{s+1}(t_s),\quad s=1,...,m(l+1)-1, \tag{7}$$

where (7) is the condition of continuity of the solution in the internal points of the partition of the interval $[0, T)$.

Let us introduce the notations $\lambda_r=x_r(t_{r-1}), r=\overline{1,m(l+1)}$, $\lambda_{m(l+1)+1}=\lim\limits_{t\to T-0}x_{m(l+1)}(t)$ and in each interval $t\in[t_{r-1},t_r)$ make a substitution $x_r(t)=u_r(t)+\lambda_r$, $r=\overline{1,m(l+1)}$. Then the boundary value problem (5) - (7) will be written as:

$$\frac{du_r}{dt}=\sum_{j=1}^{m(l+1)}\sum_{k=1}^{N}\int_{t_{j-1}}^{t_j}\tilde{\phi}_k(t)\psi_k(s)\left(u_j(s)+\lambda_j\right)ds+\tilde{f}(t),\quad t\in[t_{r-1},t_r),\quad r=\overline{1,m(l+1)} \tag{8}$$

$$u_r(t_{r-1})=0,\qquad r=\overline{1,m(l+1)} \tag{9}$$

$$\sum_{i=0}^{m}B_i\lambda_{i(l+1)+1}=d \tag{10}$$

$$\lambda_s+\lim_{t\to t_s-0}u_s(t)=\lambda_{s+1},\quad s=\overline{1,m(l+1)}. \tag{11}$$

where (11) are the conditions for continuity of the solution in the internal points of the partition Δ_N.

The solution to problem (8) - (11) is the system of pairs $(\lambda,u[t])$ with elements

$$\lambda=(\lambda_1,\lambda_2,...,\lambda_N)\in R^{nm(l+1)+1},\quad u[t]=\left(u_1(t),u_2(t),...,u_{m(l+1)}(t)\right)\in C\left([0,T],\Delta_{m(l+1)},R^{nm(l+1)}\right)$$

where functions $u_r(t)$ are continuously differentiable in $[t_{r-1},t_r), r=\overline{1,m(l+1)}$ and for

$\lambda_r = \lambda_r^*, r = \overline{1, m(l+1)}$ satisfy the system of integro-differential equations (8) and conditions (9), (10).

If the system of functions $x[t] = \left(x_1(t), x_2(t), ..., x_{m(l+1)}(t)\right)$ is a solution to problem (5) - (7), then the pair $(\lambda, u[t])$, where $\lambda = \left(x_1(t_0), x_2(t_2), ..., x_{m(l+1)}\left(t_{m(l+1)-1}\right), \lim_{t\to T-0} x_{m(l+1)}(t)\right)$, $u[t] = \left(x_1(t) - x_1(t_0), x_2(t) - x_2(t_1), ..., x_{m(l+1)}(t) - x_{m(l+1)}\left(t_{m(l+1)-1}\right)\right)$ will be a solution to problem (8) - (10). And vice versa, if the pair $\left(\tilde{\lambda}, \tilde{u}[t]\right)$ is a solution to problem (8) - (10), then the system of functions $\tilde{x}[t]$ defined by the equalities $\tilde{x}_r(t) = \tilde{\lambda}_r + \tilde{u}_r(t)$, $[t_{r-1}, t_r), r = \overline{1, m(l+1)}$, $\tilde{x}(T) = \tilde{\lambda}_{m(l+1)+1}$ is a solution to the original boundary value problem (5) - (7).

Introduction of additional parameters makes it possible to obtain the initial data (9). Now, for fixed values of parameters $\lambda \in R^{n(m(l+1)+1)}$, the system of functions $u[t]$ can be determined from the special Cauchy problem for the systems of integro-differential equations (8) and (9)

$$u_r(t) = \int_{t_r}^{t} \sum_{j=1}^{m(l+1)} \sum_{k=1}^{N} \int_{t_{j-1}}^{t_j} \tilde{\phi}_k(\tau)\psi_k(s)\left[u_j(s) + \lambda_j\right] ds d\tau + \int_{t_r}^{t} \tilde{f}(\tau) d\tau, \quad t \in [t_{r-1}, t_r), \ r = \overline{1, m(l+1)}. \quad (12)$$

Let us introduce the notation:

$$\mu_k = \sum_{j=1}^{m(l+1)} \int_{t_{j-1}}^{t_j} \psi_k(s) u_j(s) ds \quad (13)$$

and rewrite the system of integral equations (12) as:

$$u_r(t) = \int_{t_r}^{t} \sum_{k=1}^{N} \tilde{\phi}_k(\tau)\mu_k d\tau + \int_{t_r}^{t} \sum_{j=1}^{m(l+1)} \sum_{k=1}^{N} \int_{t_{j-1}}^{t_j} \tilde{\phi}_k(\tau)\psi_k(s)\lambda_j ds d\tau +$$
$$+ \int_{t_r}^{t} \tilde{f}(\tau) d\tau, \quad t \in [t_{r-1}, t_r), r = \overline{1, m(l+1)} \quad (14)$$

Multiplying both parts of (14) by $\psi_i(t)$, integrating over the interval $[t_{r-1}, t_r)$ and summing over r, we obtain a system of linear equations with respect to $\mu = (\mu_1, \mu_2, ..., \mu_N) \in R^{nN}$

$$\sum_{r=1}^{m(l+1)} \int_{t_{r-1}}^{t_r} \psi_i(\tau) u_r(\tau) d\tau = \sum_{r=1}^{m(l+1)} \int_{t_{r-1}}^{t_r} \psi_i(\tau) \int_{t_r}^{\tau} \sum_{k=1}^{N} \tilde{\phi}_k(\tau_1) d\tau_1 d\tau \mu_k +$$

$$+ \sum_{r=1}^{m(l+1)} \int_{t_{r-1}}^{t_r} \psi_i(\tau) \int_{t_r}^{\tau} \sum_{k=1}^{N} \tilde{\phi}_k(\tau_1) d\tau_1 d\tau \sum_{j=1}^{m(l+1)} \int_{t_{j-1}}^{t_j} \psi_k(s)\lambda_j ds + \sum_{r=1}^{m(l+1)} \int_{t_{r-1}}^{t_r} \psi_i(\tau) \int_{t_r}^{\tau} \tilde{f}(\tau_1) d\tau_1 d\tau$$

or

$$\mu_i = \sum_{k=1}^{N} G_{i.k}\left(\Delta_{m(l+1)}, t_{r-1}\right)\mu_k + \sum_{r=1}^{m(l+1)} V_{i.r}\left(\Delta_{m(l+1)}, t_{r-1}\right)\lambda_r + g_i\left(\Delta_{m(l+1)}, t_{r-1}\right), \quad i = \overline{1,\mathrm{N}}, \quad (15)$$

where

$$G_{i.k}\left(\Delta_{m(l+1)}, t_{r-1}\right) = \sum_{r=1}^{m(l+1)} \int_{t_{r-1}}^{t_r} \psi_i(\tau) \int_{t_{r-1}}^{\tau} \tilde{\phi}_k(\tau_1)\, d\tau_1 d\tau, \ k = \overline{1,\mathrm{N}}$$

$$V_{i.r}\left(\Delta_{m(l+1)}, t_{r-1}\right) = \sum_{j=1}^{m(l+1)} \sum_{k=1}^{N} \int_{t_{j-1}}^{t_j} \psi_i(\tau) \int_{t_{r-1}}^{\tau} \tilde{\phi}_k(\tau_1)\, d\tau_1 d\tau \int_{t_{r-1}}^{t_r} \psi_k(s) ds, \ r = \overline{1, m(l+1)}$$

$$g_i(\Delta_{m(l+1)}, t_{r-1}) = \sum_{r=1}^{m(l+1)} \int_{t_{r-1}}^{t_r} \psi_i(\tau) \int_{t_{r-1}}^{\tau} \tilde{f}(\tau_1)\, d\tau_1 d\tau .$$

Using matrices $G_{i.k}\left(\Delta_{m(l+1)}, t_{r-1}\right)$, $V_{i.r}\left(\Delta_{m(l+1)}, t_{r-1}\right)$ we can combine matrices $G\left(\Delta_{m(l+1)}, t_{r-1}\right) = \left(G_{i.k}\left(\Delta_{m(l+1)}, t_{r-1}\right)\right)$, $V\left(\Delta_{m(l+1)}, t_{r-1}\right) = \left(V_{i.r}\left(\Delta_{m(l+1)}, t_{r-1}\right)\right)$, $i = \overline{1,N}$, $r = \overline{1, m(l+1)}$ of dimensions $nN \times nN$, $nN \times nm(l+1)$ respectively, and write system (15) as

$$\left[I - G\left(\Delta_{m(l+1)}, t_{r-1}\right)\right]\mu = \sum_{r=1}^{m(l+1)} V\left(\Delta_{m(l+1)}, t_{r-1}\right)\lambda + g\left(\Delta_{m(l+1)}, t_{r-1}\right)$$

where l – is a unit matrix of nN dimension,

$$g\left(\Delta_{m(l+1)}, t_{r-1}\right) = \left(g_1\left(\Delta_{m(l+1)}, t_{r-1}\right), g_2\left(\Delta_{m(l+1)}, t_{r-1}\right), \ldots, g_N\left(\Delta_{m(l+1)}, t_{r-1}\right)\right) \in R^{nN} .$$

Definition 1. Partitioning $\Delta_{m(l+1)}$ is called regular if the matrix $I - G\left(\Delta_{m(l+1)}, t_{r-1}\right)$ is reversible.

Let $\sigma\left(\mathrm{N}, [0,T]\right)$ denote a set of regular partitions $\Delta_{m(l+1)}$ from $[0,T]$ for equation (5).

Definition 2. The special Cauchy problem (8), (9) for a Fredholm integro-differential equation is called uniquely solvable if for any $\lambda \in R^{n(m(l+1)+1)}$, $\tilde{f}(t) \in C\left([0,T], \Delta_{m(l+1)}, R^n\right)$ it has a unique solution.

The special Cauchy problem (8), (9) is equivalent to the system of integral equations (12). Due to the degeneracy of the kernel, this system is equivalent to the system of algebraic equations (15) with respect to $\mu = \left(\mu_1, \mu_2, \ldots, \mu_N\right) \in R^{nN}$. Therefore, the special Cauchy problem is uniquely solvable if and only if the partition $\Delta_{m(l+1)}$ generating this problem is regular. Since the special Cauchy problem is uniquely solvable for sufficiently large $l \in \mathrm{N}$, the set $\sigma\left(\mathrm{N}, [0,T]\right)$ is not empty.

Let us take $\Delta_{m(l+1)} \in \sigma(N,[0,T])$ and represent $\left[I - G\left(\Delta_{m(l+1)}, t_{r-1}\right)\right]^{-1}$ as $\left[I - G\left(\Delta_{m(l+1)}, t_{r-1}\right)\right]^{-1} = M_{k.i}\left(\Delta_{m(l+1)}, t_{r-1}\right)$, $k,i=\overline{1,\mathrm{N}}$, where $M_{k.i}\left(\Delta_{m(l+1)}, t_{r-1}\right)$ are square matrices of dimension nN. Then, according to (15), the elements of the vector $\mu \in R^{nN}$ can be determined by the equalities

$$\mu_i = \sum_{j=1}^{m(l+1)} \sum_{p=1}^{N} M_{i.p}\left(\Delta_{m(l+1)}, t_r\right) V_{i.j}\left(\Delta_{m(l+1)}, t_r\right)\lambda_j + \sum_{p=1}^{N} M_{i.p}\left(\Delta_{m(l+1)}, t_r\right)\cdot g_p\left(\Delta_{m(l+1)}, t_r\right), \quad i=\overline{1,N} \qquad (16)$$

Substituting the right-hand side of (16) into equality (13), we obtain functions $u_r(t)$ in terms of λ_j, $j=\overline{1,m(l+1)}$.

$$u_r\left(\Delta_{m(l+1)}, t_{r-1}, t\right) = \sum_{j=1}^{m(l+1)} \sum_{p=1}^{N} \int_{t_r}^{t} \tilde{\varphi}_k(\tau)\, d\tau \left[\sum_{p=1}^{N} M_{k.p}\left(\Delta_{m(l+1)}, t_{r-1}\right)\cdot V_{p.j}\left(\Delta_{m(l+1)}, t_{r-1}\right) + \int_{t_{j-1}}^{t_j} \psi_k(s)\, ds\right]\lambda_j + \sum_{k=1}^{N} \int_{t_r}^{t} \tilde{\varphi}_k(\tau)\, d\tau \sum_{p=1}^{N} M_{k.p}\left(\Delta_{m(l+1)}, t_{r-1}\right)\cdot g_p\left(\Delta_{m(l+1)}, t_{r-1}\right) + \int_{t_r}^{t} \tilde{f}(\tau)\, d\tau, \qquad (17)$$

$$t, t_r = \left[t_{r-1}, t_r\right), \quad r=\overline{1,m(l+1)}$$

Let us introduce the notations:

$$D_{r,j}\left(\Delta_{m(l+1)}, t_{r-1}, t\right) = \sum_{k=1}^{N} \int_{t_{r-1}}^{t} \tilde{\varphi}_k(\tau) d\tau \left[\sum_{p=1}^{N} M_{k.p}\left(\Delta_{m(l+1)}, t_{r-1}\right) V_{p,j}\left(\Delta_{m(l+1)}, t_{r-1}\right) + \int_{t_{j-1}}^{t_j} \psi_k(s)\, ds\right],$$

$$r, j = \overline{1,m(l+1)} \qquad (18)$$

$$F_r\left(\Delta_{m(l+1)}, t_{r-1}, t\right) = \sum_{k=1}^{N} \int_{t_{r-1}}^{t} \tilde{\varphi}_k(\tau) \sum_{p=1}^{N} M_{k.p}\left(\Delta_{m(l+1)}, t_{r-1}\right)\cdot g\left(\Delta_{m(l+1)}, t_{r-1}\right) + \int_{t_r}^{t} \tilde{f}(\tau)\, d\tau. \qquad (19)$$

Then formula (2.17) will be written as

$$u_r\left(\Delta_{m(l+1)}, t_{r-1}, t\right) = \sum_{j=1}^{m(l+1)} D_{rj}\left(\Delta_{m(l+1)}, t_{r-1}, t\right)\lambda_j + F_r\left(\Delta_{m(l+1)}, t_{r-1}, t\right), \quad r=\overline{1,m(l+1)}. \qquad (20)$$

Having determined the limiting values $\lim\limits_{t\to t_r-0} u_r(t)$, $r=\overline{1,m(l+1)}$, we substitute them into the boundary continuity condition (11). Then we obtain the following system of linear algebraic equations in parameters λ_r, $r=\overline{1,m(l+1)+1}$:

$$\sum_{i=0}^{m} B_i \lambda_{i(l+1)+1} = d \qquad (21)$$

$$\lambda_s + \sum_{j=1}^{m(l+1)} D_{sj}\left(\Delta_{m(l+1)}, t_{s-1}, t_s\right)\lambda_j - \lambda_{s+1} = -F_s\left(\Delta_{m(l+1)}, t_{s-1}, t_s\right), \ \ s = \overline{1, m(l+1)}\ . \qquad (22)$$

If we denote the matrix corresponding to the left-hand side of the system of equations (22), (23) as $Q_*\left(\Delta_{m(l+1)}\right)$, then the system can be written in matrix form:

$$Q_*\left(\Delta_{m(l+1)}\right)\cdot\lambda = -F_*\left(\Delta_{m(l+1)}, t_{r-1}\right), \quad \lambda \in R^{n(m(l+1)+1)} \qquad (23)$$

where $F_*\left(\Delta_{m(l+1)}\right) = \left(-d\,,\, F_1\left(\Delta_{m(l+1)}, t_1\right), \ldots, F_{m(l+1)}\left(\Delta_{m(l+1)}, T\right)\right)$.

Definition 3. Problem (3), (4) is called uniquely solvable if for any pair $\left(f(t), d\right)$ it has a unique solution $x(t)$.

From the above, it follows:

Theorem 1. Let the matrix $\left[I - A^2\right]$ be not degenerate, then the boundary value problem (1), (2) is uniquely solvable if and only if the matrix $Q_*\left(\Delta_{m(l+1)}\right)$ is invertible for any $\Delta_{m(l+1)} \in \sigma\left(N, [0, T]\right)$.

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REFERENCES

1 Brunner H. Collocation Methods for Volterra Integral and Related Functional Equations. Cambridge: Cambridge University Press; 2004.

2 Maleknejad K, Attary M. An efficient numerical approximation for the linear Fredholm integro-differential equations based on Cattani's method. Commun Nonlinear Sci Numer Simulat. -2011. -Vol.16:2672-2679.

3 Molabahrami A. Direct computation method for solving a general nonlinear Fredholm integro-differential equation under the mixed conditions: degenerate and non-degenerate Kernels/ J Comput Appl Math. -2015.- Vol.282:34-43.

4 Turmetov B.Kh., Usmanov K.I., Nazarova K.Zh., On the operator method for solving linear integro-differential equations with fractional conformable derivatives // Fractal and Fractional, -2021. No. 5(3), p.109;

5 Dzhumabayev D.S. Criteria for the unique solvability of a linear boundary-value problem for an ordinary differential equation// Computational Mathematics and Mathematical Physics. -1989. -Vol.29, No. 1.- P.34-46.

6 Dzhumabaev D.S. A method for solving a linear boundary value problem for an integro-differential equation // Jrn. Comp. Mat. and Mat. Phys.,- 2010. -V. 50, No. 7. P. 1209-1221.

7 D. S. Dzhumabaev, "On one approach to solve the linear boundary value problems for Fredholm integro-differential equations", Journal of Computational and Applied Mathematics, 294:2 (2016), 342-357.

8 Dulat Dzhumabaev, "Computational methods of solving the boundary value problems for the loaded differential and Fredholm integro-differential equations", Mathematical Methods in Applied Sciences, 41:4 (2018), 1439-1462

9 Nazarova K.Zh., Usmanov K.I. Unique solvability of the boundary value problem for integro-differential equations with involution // AIP Conference Proceedings. 2365, 070012 (2021);

10 K. Nazarova, K. Usmanov., On a boundary value problem for systems of integro-differential equations with involution // International Journal of Applied Mathematics -2021. –Vol.34. -P.225-235.

11 Orumbayeva N.T. Algorithms for finding a solution to the initial boundary value problems for differential equations in partial derivatives / N.T. Orumbayeva // Bulletin of the Karaganda University-Mathematics. -2016. - No. 2 (82). --P. 107–112.

12 Przeworska-Rolewicz D. Equations with Transformed Argument, An Algebraic Approach. Amsterdam, Warszawa,1973.

13 Wiener J. Generalized Solutions of Functional Differential Equations. World Sci., Singapore, New Jersey, London, Hong Kong, 1993.

14 Fite W.B. Properties of the solution of certain functional differential equations // Trans. Amer. Math. Soc. - 1921. - No. 3. - P. 311–319.

15 Litvinchuk G.S. Boundary value problems and singular integral equations with a shift. - Publishing house "Nauka", Main edition of physical and mathematical literature, 1977.

16 Karapetyants N.K., Samko S.G. Equations with involution operators and their applications // Rostov-n / D. Publishing house of RSU -1988. 188 p.

17 Kritskov L.V., Sadybekov M.A., Sarsenbi A.M. Properties in Lp of root functions for a nonlocal problem with involution// Turk J Math. – 2019. - V.43. –P.393 - 401.

18 Sadybekov M.A., Sarsenbi A.M. Criterion for the basis property of the eigenfunction system of a multiple differentiation operator with an involution// Differential Equation. -2012. -Vol.48, No.8. -P. 1112 - 1118.

19 Cabada A, Tojo FAF. On linear differential equations and systems with reflection// Applied Mathematics and Computation. - 2017. -Vol.305, -P. 84-102.

20 Tojo FAF. Computation of Green's functions through algebraic decomposition of operators// Boundary Value Problem. - 2016 . –Vol. 2016, No. 167. - P. 1-15.

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ИНВОЛЮЦИЯСЫ БАР ИНТЕГРАЛДЫҚ-ДИФФЕРЕНЦИАЛДЫҚ ТЕҢДЕЛЕР ЖҮЙЕЛЕРІ ҮШІН КӨП НҮКТЕЛІ ШЕТТІК ЕСЕПТІҢ БІРМӘНДІ ШЕШІЛІМДІЛІГІНІҢ КРИТЕРИЯСЫ

[0, T] кесіндісінде интегралдық мүшенің ерекше болған жағдайда инволютивтік түрлендіруі бар интегралдық-дифференциалдық теңдеулер жүйелері үшін көп нүктелі шеттік есеп қарастырылады. Инволюция қасиетін пайдалана отырып, бастапқы есеп ерекше ядролы интегралдық-дифференциалдық теңдеулер жүйелері үшін көп нүктелі шеттік есептерге келтіріледі. Параметрлерді енгізу және айнымалыларды алмастыру, сонымен қатар ядроның ерекшелігін пайдалану арқылы бастапқы есептің бірмәнді шешілімділігі бастапқы берілімдер мәндерінен тәуелді матрицаның кері матрицасының бар болуына келтіріледі.

***Түйін сөздер**: шеттік есеп, интегралдық-дифференциалдық теңдеулер жүйесі, инволюция, инволютивтік түрлендірулер, параметрлеу әдісі.*

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КРИТЕРИЙ ОДНОЗНАЧНОЙ РАЗРЕШИМОСТИ МНОГОТОЧЕЧНОЙ КРАЕВОЙ ЗАДАЧИ ДЛЯ СИСТЕМ ИНТЕГРО-ДИФФЕРЕНЦИАЛЬНЫХ УРАВНЕНИЙ С ИНВОЛЮЦИЕЙ

На отрезке [0, T] рассматривается многоточечная краевая задача для систем интегро-дифференциальных уравнений с инволютивным преобразованием, когда ядро интегрального члена является вырожденным. Используя свойство инволюции, задача сводится к многоточечной краевой задаче для систем интегро-дифференциальных уравнений с вырожденным ядром. Введя параметры и выполняя замену переменных, а также используя вырожденность ядра, однозначная разрешимость исходной задачи сводится к обратимости матрицы, зависящего от исходных данных.

Ключевые слова. *краевая задача, система интегро-дифференциальных уравнений, инволюция, инволютивные преобразования, метод параметризации.*