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N. M. TEMIRBEKOV[1]*, S. I. KABANIKHIN[2], L. N. TEMIRBEKOVA[3], ZH. E. DEMEUBAYEVA[4]

[1]National engineering academy Republic of Kazakhstan, Almaty, Kazakhstan,
e-mail: temirbekov@rambler.ru
[2]Institute of Computational Mathematics and Mathematical Geophysics SB RAS, RF
e-mail: ksi52@mail.ru
[3]Abay Kazakh National Pedagogical University, Almaty, Kazakhstan
e-mail: laura-nurlan@mail.ru
[4]D. Serikbayev East Kazakhstan Technical University, Ust-Kamenogorsk, Kazakhstan
e-mail: demeubayevazhanar@gmail.com

GELFAND-LEVITAN INTEGRAL EQUATION FOR SOLVING COEFFICIENT INVERSE PROBLEM

In this paper, numerical methods for solving multidimensional equations of hyperbolic type by the Gelfand-Levitan method are proposed and implemented. The Gelfand-Levitan method is one of the most widely used in the theory of inverse problems and consists in reducing a nonlinear inverse problem to a one-parameter family of linear Fredholm integral equations of the first and second kind. In the class of generalized functions, the initial-boundary value problem for a multidimensional hyperbolic equation is reduced to the Goursat problem. Discretization and numerical implementation of the direct Goursat problem are obtained to obtain additional information for solving a multidimensional inverse problem of hyperbolic type. For the numerical solution, a sequence of Goursat problems is used for each given y. A comparative analysis of numerical experiments of the two-dimensional Gelfand-Levitan equation is performed. Numerical experiments are presented in the form of tables and figures for various continuous functions q(x, y).

***Key words**: inverse problem, direct problem, hyperbolic type, Gelfand-Levitan equation, Goursat problem, Numerical solution.*

Introduction. The Gelfand-Levitan method is one of the most widely used in the theory of inverse problems and consists in reducing a nonlinear inverse problem to a one-parameter family of linear Fredholm integral equations of the first and second kind. Let us briefly describe the achievements of scientific research in this area.

In work, Gelfand I.M. and Levitan B.M. [1] proposed a method for reconstructing the Sturm-Liouville operator from a spectrum function and provided sufficient conditions for a given monotone function to be a spectral function of the operator. Krein M.G. [2] considered the physical formulation of the string tension problem and theorems on the solution of the inverse boundary value problem. Blagoveshchensky A.S. [3] provided the new evidence on the theory of inverse problems for the string equation. The advantage of the new proof is that it is simple and local (non-stationary).

A detailed review of numerical methods for solving equations of the Gelfand-Levitan type is given in the work of Pariyskiy B.S. [4].

* E-mail корреспондирующего автора: temirbekov@rambler.ru

In the monograph of Kabanikhin S.I. [5] proposed a new algorithm for solving the Gelfand-Levitan equation, which involves the use of a sufficient condition for the solvability of the inverse problem.

In the monograph of Romanov and Kabanikhin S.I. [6,7] a dynamic version of the Gelfand-Levitan method is presented as applied to a one-dimensional inverse geoelectric problem for the quasi-stationary approximation of Maxwell's equations.

Multidimensional Gelfand-Levitan equations were obtained in the works of Belishchev, Kabanikhin S.I., Blagoveshchensky A.S. [8-10].

In the works of Bakanov G.B. [11-13] considered a discrete analog of the Gelfand-Levitan method for a two-dimensional inverse problem of hyperbolic type.

In [14], gradient and direct methods for solving the Gelfand-Levitan equations were numerically implemented.

One-dimensional and multidimensional methods for solving inverse problems for the wave equation by the Gelfand-Levitan method lead to the numerical solution of Fredholm integral equations of the first and second kind. In the work of Lavrentiev M.M. [16] considered integral equations of the first kind.

In recent years, there has been a growing interest in approximate methods for solving integral equations. There are several numerical methods for solving integral equations of the first kind. Most of the works are based on projection methods, such as the Galerkin–Petrov method, the Bubnov–Galerkin method, the method of moments, and the collocation method [17]. One of the most attractive developments in recent years has been the use of wavelets as basis functions in projection methods. The wavelet technique allows to create very efficient algorithms compared to known regularizing algorithms. Various wavelet bases are used in the papers [17-22].

Statement and solution of the two-dimensional coefficient inverse problem. We consider a sequence of direct problems [2]

$$u_{tt}^{(k)} = u_{xx}^{(k)} + u_{yy}^{(k)} + q(x,y)u^{(k)}, \ \ x>0, \ \ y\in[-\pi,\pi], \ \ t\in R, \ \ k\in Z, \tag{1}$$

$$u^{(k)}\,|_{t=0} = 0, \tag{2}$$

$$u_t^{(k)}\,|_{t=0} = h(y)\delta(x), \tag{3}$$

$$u^{(k)}\,|_{y=\pi} = u^{(k)}\,|_{y=-\pi}, \tag{4}$$

We assume that the trace of the solution of the direct problem (1) - (4) exists and can be measured. In the inverse problem, it is required to restore a continuous function $q(x, y)$ from additional information about the solution of the direct problem

$$u^{(k)}(0,y,t) = f^{(k)}(y,t)\ , \ y\in(-\pi,\pi), \ \ t>0, \ \ k\in Z \tag{5}$$

where R is the set of real numbers, Z is the set of all integers, δ is the Dirac delta function, k is some fixed integer, $h(y)=e^{iky}$. Here and everywhere below, we assume that all the considered functions are sufficiently smooth and 2π – periodic in the variable y.

The necessary condition for the existence of a solution (1)-(5) is as follows:

$$f^{(k)}(y,0) = 0\ .$$

The generalized solution of the direct problem (1)-(4) is a piecewise continuous solution of the integral equation

$$u^{(k)}(x,y,t)=\frac{h(y)}{2}\theta(t-|x|)-\frac{1}{2}\iint\limits_{\Delta(x,y,t)} q(\xi,y)u^{(k)}(\xi,y,\tau)\,d\xi d\tau\ . \tag{6}$$

Here θ(t) – the Heaviside theta function.

By analogy with the one-dimensional case, it follows from the integral equation (6) that

$$u(x,y,t)=0,\quad t<|x|,\quad (x,t)\in R\times R_+ \tag{7}$$

For $t>|x|$ we have the formula

$$u^{(k)}(x,y,t)=\frac{h(y)}{2}-\frac{1}{2}\iint\limits_{(x,y,t)} q(\xi,y)u^{(k)}(\xi,y,\tau)\,d\xi d\tau, \tag{8}$$

here $(x,y,t)=\{(\xi,y,\tau):|\xi|\le\tau\le t-|x-\xi|\}$.

From formula (8) it follows that

$$u^{(k)}(x,y,|x|)=\frac{h(y)}{2}\ . \tag{9}$$

Thus, to solve the direct problem in the class of generalized functions, we have the Goursat problem (1), (9) which determines the classical solution of problem (1) - (4) [2].

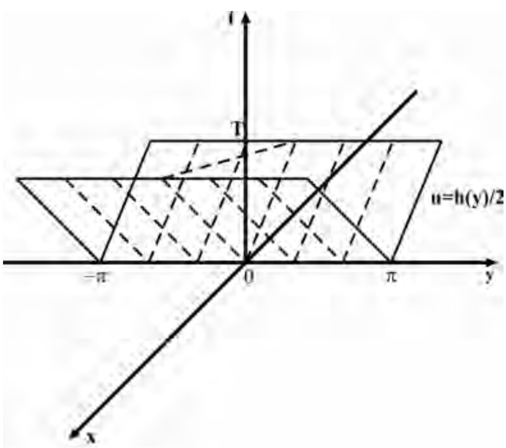

Figure 1 – Scheme for solving the direct problem (1), (9)

A sequence of auxiliary direct problems is introduced [2]:

$$\omega_{tt}^{(m)}=\omega_{xx}^{(m)}+\omega_{yy}^{(m)}+q(x,y)\omega^{(m)},\ \ x>0,\ \ y\in[-\pi,\pi],\ \ t\in R,\ \ m\in Z. \tag{10}$$

$$\omega^{(m)}(0,y,t)=e^{imy}\delta(t),\ \ \frac{\partial\omega^{(m)}}{\partial x}(0,y,t)=0\ , \tag{11}$$

$$\omega^{(m)}\big|_{y=\pi} = \omega^{(m)}\big|_{y=-\pi} \quad . \tag{12}$$

where

$$\tilde{\omega}^{(m)}(x, y, x-0) = \frac{h(y)}{4}\int_0^x q(\xi, y)\,d\xi, \;\; x > 0 \tag{13}$$

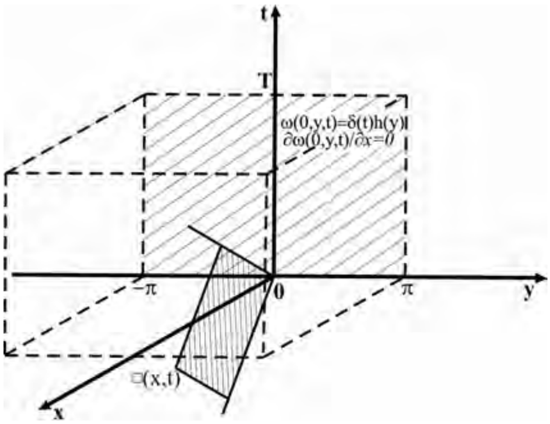

Figure 2 – Area for solving the inverse problem (1) - (5)

Functions $u^{(k)}(x,y,t)$ and $f^{(k)}(y,t)$ with respect to the variable *t*. For $x > |t|$ we have

$$\frac{1}{2}\left[f^{(k)}(y,t+x) + f^{(k)}(y,t-x)\right] + \int_{-x}^{x}\sum_{m=1}^{\infty} f_m^{(k)}(t-s)\tilde{\omega}^{(m)}(x,y,s)ds = 0 \;. \tag{14}$$

For each fixed $x > 0$ relation (14) is an integral equation of the first kind with respect to the function $\tilde{\omega}(x,y,t)$, $t \in (-x, x)$. Equation (14) is the Gelfand-Levitan equation.

Discretization of the two-dimensional Gelfand–Levitan equation. In the Gelfand-Levitan equations (14), we replace the integral by the sum and for $t = t_j$, $j = -N, -N+1, ..., 0, ..., N-1, N$ we obtain a system consisting of (2*N* + 1) equations with $M \times (2N+1)$ unknowns $\tilde{\omega}^{(m)}(x,y,s_i)$, $m = 1, 2, ..., M$; $i = -N, -N+1, ..., N-1, N$;

$$\sum_{m=1}^{M}\sum_{i=-N}^{N} f_m^{(k)}(t_j - s_i)\tilde{\omega}^{(m)}(x,y,s_i)\tau = -\frac{1}{2}[f^{(k)}(y,t_j+x) + f^{(k)}(y,t_j-x)] \,. \tag{15}$$

Equation (15) in matrix form can be written in the following form

$$\sum_{m=1}^{M} F_m \vec{\tilde{\omega}}^{(m)} = \vec{f}^{(k)} \quad , \tag{16}$$

where $F_m = \{f_m^{(k)}(t_j - s_i)\}_{\substack{i=-\overline{N,N},\\ j=-\overline{N,N}.}}$, *m* = 1, 2, ..., *M* are square matrices of size 2*N* + 1.

Search vectors and right part

$$\vec{\tilde{\omega}}^{(m)} = \{\tilde{\omega}^{(m)}(x, y, s_i)\}_{i=\overline{-N,N}}, \; m = 1, 2, ..., M. \quad (17)$$

$$\vec{f}^{(k)} = \{-0.5(f^{(k)}(y, t_j + x) + f(y, t_j - x))\}_{j=\overline{-N,N}} . \quad (18)$$

Further, we assume that $m = 1$, then formula (4) takes the form

$$\sum_{i=-N}^{N} f^{(k)}(t_j - s_i)\tilde{\omega}(x, y, s_i)\tau = -\frac{1}{2}[f^{(k)}(y, t_j + x) + f^{(k)}(y, t_j - x)] . \quad (19)$$

Let us rewrite equation (19) in operator form

$$A^{(k)}\tilde{\omega} = f^{(k)} . \quad 20)$$

For the numerical solution of the Gelfand-Levitan equation (19), the simple iteration method (in the theory of ill-posed problems, the Landweber iteration method) is used in combination with M.M. Lavrentiev regularization. Equation (20) is replaced by the following correct equation

$$\left(\mu E + A^{(k)}\right)\tilde{\omega} = f_{\gamma}^{(k)} \quad (21)$$

where $f_{\gamma}^{(k)} = f^{(k)} + \mu\tilde{\omega}_0$, E is an identity matrix, μ is a positive parameter of M.M. Lavrentiev's regularization, $\tilde{\omega}_0$ is a trial solution, i.e. some approximation to the desired solution.

Iterative process with regularization M.M. Lavrentiev will take the form

$$\frac{\tilde{\omega}_{n+1} - \tilde{\omega}_n}{\tau} + (\mu E + A^{(k)})\tilde{\omega}_n = f_{\gamma}^{(k)} .$$

The iteration calculation algorithm is as follows [2]:

1. The initial approximation is set equal to the right side $f_{\gamma}^{(k)}$;

2. The accuracy of the calculation ε is set, for the condition of the end of the iterative process $\left|\tilde{\omega}_{n+1} - \tilde{\omega}_n\right| < \varepsilon$;

3. The calculation is carried out according to the following iterative process

$$\frac{\tilde{\omega}_{n+1} - \tilde{\omega}_n}{\tau} + (\mu E + A^{(k)})\tilde{\omega}_n = f_{\gamma}^{(k)} .$$

The works of many authors [3-10] are devoted to the numerical method for solving coefficient inverse problems for hyperbolic equations.

Based on the Landweber iteration method for solving a two-dimensional coefficient inverse problem of source recovery $q(x, y)$, an efficient algorithm for numerical implementation was developed and a program code was written. During a series of numerical experiments, various functions $q(x, y)$ were taken, which will be given below. The initial approximation was chosen to be equal to the right side of the system of linear equations. Numerical calculations have been made to find the desired function $\tilde{\omega}\ (x, y, t)$, $x \in (0, x_L)$, $y \in (0, x_L)$ and to restore the coefficient $q(x, y)$ from it.

During a series of numerical experiments, various functions $q(x, y)$ were taken.

Figures 3, 4 show the results of numerical calculations by the **Landweber iteration method** for $e = 0{,}001$ of the function

$$q(x,y) = \frac{e}{(x-0.51)^2 + (y-0.51)^2},$$

$$\tilde{\omega}(x,y,x-0) = \frac{e}{4(y-0.51)}\left(arctg\frac{x-0.51}{y-0.51} - arctg\frac{-0.51}{y-0.51} \right)$$

which is found by formula (33). During the calculation in this example, the number of layers n was taken equal to 10, the regularization parameter $\mu = 0{,}5$, $x_L = 1$, $\varepsilon = 0{,}000001$ and the following output data were obtained: error $\left\| \tilde{\omega}_{н} - \tilde{\omega}_{пр} \right\| = 0{,}00031857757$, number of iterations 245, amount of computer time 6.78 sec.

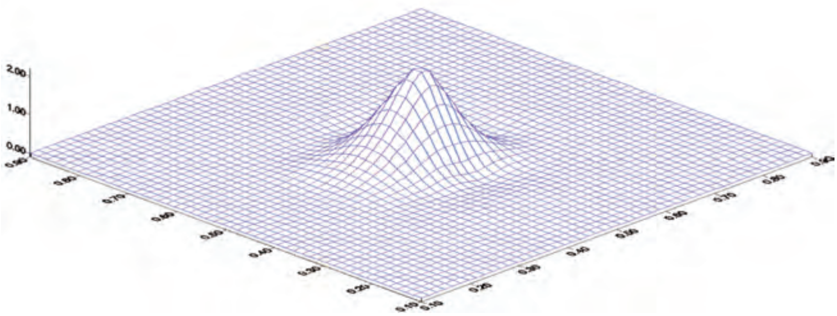

Figure 3 – Graph of the approximate solution of $q(x, y)$ restored by the Landweber method, at $n = 10$, $e = 0{,}001$

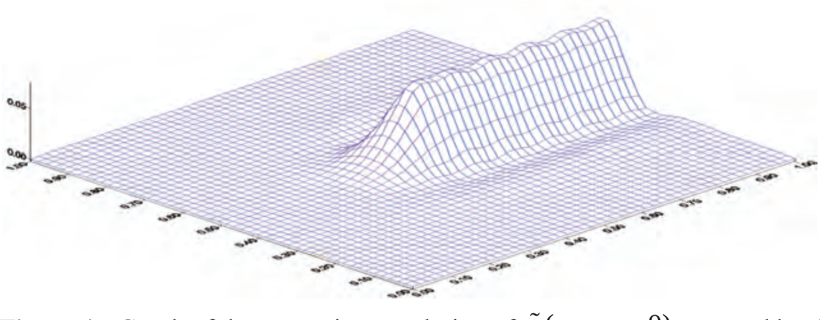

Figure 4 – Graph of the approximate solution of $\tilde{\omega}(x,y,x-0)$ restored by the Landweber iteration method, at $n = 10$, $e = 0{,}001$

The results of numerical calculations by the **method of conjugate gradients** for $e = 0{,}001$ the functions $q(x, y)$, $\tilde{\omega}(x,y,x-0)$, which are given above. The number of layers $n = 10$, the regularization parameter $\mu = 0{,}5$, $x_L = 1$, $\varepsilon = 0{,}000001$ and the following data were obtained: error $\left\| \tilde{\omega}_{н} - \tilde{\omega}_{пр} \right\| = 0{,}000004144269$, the number of iterations 3, the amount of computer time 29.25 sec.

Numerical calculations are carried out using the **square root method** for e = 0,001 the function $q(x, y)$. $\tilde{\omega}(x, y, x-0)$, which are given above. The number of layers n = 10, the regularization parameter μ = 0,5, x_L = 1, ε = 0,000001 and the following data were obtained: error $\left\|\tilde{\omega}_{\scriptscriptstyle T} - \tilde{\omega}_{\text{пр}}\right\| = 0{,}000000014901161$, the amount of computer time 4.05 sec.

Table 1 – Comparative analysis of the numerical solution of the two-dimensional GLE equation by various methods with the number of grid nodes n = 10, μ = 0,5, ε = 0,000001 for the function $q(x, y)$ where e = 0,001

Methods	Number of iterations	Error rate $\left\|\tilde{\omega}_T - \tilde{\omega}_{\text{пр}}\right\|$	Amount of computer time	Convergence
Landweber iteration method	245	0,00031857757	6,78 sec.	converges
Method of conjugate gradients	3	0,000004144269	29,25 sec.	converges
Quadratic root method	-	0,0000000149011	4,05 sec.	converges

The table shows that the results of the numerical solution of problem (17) - (21) by various numerical methods show high accuracy.

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Н. М. ТЕМІРБЕКОВ, С. И. КАБАНИХИН, Л. Н. ТЕМІРБЕКОВА, Ж. Е. ДЕМЕУБАЕВА

Қазақстан Республикасы Ұлттық инженерлік академиясы, Алматы, Қазақстан,
e-mail: temirbekov@rambler.ru
РҒА СБ Есептік математика және математикалық геофизика институты, РФ
e-mail: ksi52@mail.ru
Абай атындағы Қазақ ұлттық педагогикалық университеті, Алматы, Қазақстан

e-mail: laura-nurlan@mail.ru
Д. Серікбаев атындағы Шығыс Қазақстан техникалық университеті, Өскемен, Қазақстан
e-mail: demeubayevazhanar@gmail.com

ГЕЛЬФАНД-ЛЕВИТАН ИНТЕГРАЛДЫҚ ТЕҢДЕУІН ҚОЛДАНЫП КЕРІ КОЭФИЦИЕНТТІК ЕСЕПТІ ШЕШУ

Бұл жұмыста Гельфанд-Левитан әдісімен гиперболалық типті көп өлшемді теңдеулерді шешудің сандық әдістері ұсынылған және жасалған. Гельфанд-Левитан әдісі кері есептер теориясында кеңінен қолданылатын әдіс болып табылады және сызықты емес кері есепті бірінші және екінші текті сызықтық Фредгольм интегралдық теңдеулерінің бір параметрлі тобына келтіруден тұрады. Жалпыланған функциялар класында көпөлшемді гиперболалық теңдеу үшін бастапқы-шектік есеп Гурса есебіне келтіріледі. Гиперболалық типті көп өлшемді кері есепті шешу үшін қосымша ақпарат алу үшін тура Гурса есебінің дискретизациясы және сандық шешімі алынған. Сандық шешім үшін әр берілген у Гурса есептерінің тізбегі пайдаланылады. Екі өлшемді Гельфанд-Левитан теңдеуінің сандық тәжірибелеріне салыстырмалы талдау жасалды. Сандық тәжірибелер әртүрлі үздіксіз q(x, y) функциялар үшін кестелер мен суреттер түрінде ұсынылған .

***Түйін сөздер**: кері есеп, тура есеп, гиперболалық тип, Гельфанд-Левитан теңдеуі, Гурсат есебі, сандық шешім.*

Н. М. ТЕМИРБЕКОВ, С. И. КАБАНИХИН, Л. Н. ТЕМИРБЕКОВА, Ж. Е. ДЕМЕУБАЕВА

Национальная инженерная академия Республики Казахстан, г. Алматы, Казахстан,
e-mail: temirbekov@rambler.ru
Институт вычислительной математики и математической геофизики СО РАН, РФ
e-mail: ksi52@mail.ru
Казахский национальный педагогический университет имени Абая, Алматы, Казахстан
e-mail: laura-nurlan@mail.ru
Восточно-Казахстанский технический университет имени Д,Серикбаева, Усть-Каменогорск, Казахстан
e-mail: demeubayevazhanar@gmail.com

ИНТЕГРАЛЬНОЕ УРАВНЕНИЕ ГЕЛЬФАНДА-ЛЕВИТАНА ДЛЯ РЕШЕНИЯ КОЭФФИЦИЕНТНОЙ ОБРАТНОЙ ЗАДАЧИ

В работе предложены и реализованы численные методы решения многомерных уравнений гиперболического типа методом Гельфанда-Левитана. Метод Гельфанда-Левитана является одним из наиболее широко используемых в теории обратных задач и заключается в сведении нелинейной обратной задачи к однопараметрическому семейству линейных интегральных уравнений Фредгольма первого и второго рода. В классе обобщенных функций начально-краевая задача для многомерного гиперболического уравнения сводится к задаче Гурса. Получены дискретизация и численная реализация прямой задачи Гурса для получения дополнительной информации для реше-

ния многомерной обратной задачи гиперболического типа. Для численного решения используется последовательность задач Гурса для каждого заданного y. Проведен сравнительный анализ численных экспериментов двумерного уравнения Гельфанда-Левитана. Численные эксперименты представлены в виде таблиц и рисунков для различных непрерывных функций q(x, y).

***Ключевые слова**: обратная задача, прямая задача, гиперболический тип, уравнение Гельфанда-Левитана, задача Гурса, численное решение.*