UDC 517.51: 087.57
https://doi.org/10.47533/2020.1606-146X.178

R. MOLDASHEVA[1]*, A. ISMAILOVA[1], A. ZADAGALI[2]

[1]S. Seifullin Kazakh Agrotechnical University, Nur-Sultan, Kazakhstan
[2]L. N. Gumilyov Eurasian National University, Nur-Sultan, Kazakhstan
raushan85_07@mail.ru, a.ismailova@mail.ru, z.a.aizhan1993@gmail.com

SIGNS OF STABILITY OF AQUATIC ECOSYSTEMS IN MATHEMATICAL MODELS

To date, various data on water resources have been accumulated, but hydrobiological and hydrochemical indicators remain available to assess the current state of aquatic ecosystems, which can be the basis for assessing the environmental situation within the water body. Systematization of multi-year and diverse data on the lakes and rivers of the country, the use of mathematical tools for assessing and forecasting the state of the aquatic ecosystem is impossible without the use of information and communication technologies. Quality mathematical modeling of aquatic ecosystems and the development of information and analytical system for the study of aquatic ecosystems is an urgent task, including databases of various-quality data on the water body and its ecosystem, data management and processing tools, as well as a set of mathematical models for the functioning of the water body ecosystem.

Research is based on information technology, statistical data processing, and mathematical modeling. Mathematical models are based on systems of differential equations, solutions are sought with the help of own computing programs and software suites (Maple, Matlab, Mathematics, etc.). When possible, modeling includes analytical studies of the properties of solutions, primarily this concerns stationary or spatially homogeneous solutions, as well as asymptotic properties of solutions. The lower trophic levels of the water body ecosystem are studied, as this determines the functioning of aquatic ecosystems. The species composition of phytoplankton is an indicator of the ecological state of the water body. Based on the quantitative characteristics of phytoplankton, the bioproductivity of the aquatic ecosystem is calculated. The physical and chemical characteristics of water allow drawing conclusions about the pollution of the water body and the composition of mineral nutrition for phytoplankton.

***Key words**: mathematical modeling, information-analytical system, aquatic ecosystem, database, phytoplankton*

Introduction. At present, all aquatic ecosystems are subject to varying degrees of anthropogenic stress due to global processes taking place on the planet. The analysis of the effects of such an influence is difficult but important for the development and change of the natural system. Using mathematical models is reasonable to carry out such an analysis.

The first exciting scientific results in mathematical ecology were obtained in the twenties and thirties of the last century: it should not go unmentioned the works of A.V. Kostitsyn [1], related to hydrobiology, and as of the second half of the twentieth century - the works of S.E. Jorgensen, M. Strashcraba [2], considering the issues of mathematical modeling of freshwater ecosystems, V.V. Menshutkin [3] – considering the issues of simulation modeling of aquatic ecosystems, the works of the scientific school of the academician I.I. Vorovich [4], devoted to the mathematical modeling of ecological and economic systems, etc. In general, the considered works developed conceptual approaches to the mathematical description of the processes of life of aquatic organisms and aquatic ecosystems commu-

* E-mail корреспондирующего автора: raushan_mn@mail.ru

nities (descriptive models), qualitative models (clarifying the dynamic mechanism of the studied process and making it possible to reproduce the observed dynamic effects in the behavior of the system); simulation models of specific complex systems, taking into account all available information about the object (and allowing to predict the behavior of systems or solve the optimization problems of their operation) [5].

Modeling is one of the most important methods of scientific cognition, using which a model (conditional image) of the research object is built. Its essence is that the relationship of the studied phenomena and factors is presented in the form of specific mathematical equations. The process of building a mathematical model includes the following typical stages [6]:

– formulation of the modeling objectives;

– qualitative analysis of the ecosystem based on these objectives;

– formulation of laws and plausible hypotheses regarding the structure of the ecosystem, the mechanisms of its behavior in general or individual parts (when self-organized, coputer "finds" these laws);

– identification of the model (determination of its parameters);

– verification of the model (verification of its functionality and assessment of the degree of appropriateness to a real ecosystem);

– study of the model (analysis of the stability of its solutions, sensitivity to changes in parameters, etc.) and experiment with it.

Many processes in water bodies have not yet been studied in sufficient detail. The processes of study continue, followed by and together with meaningful cognition there comes information support and formal analysis in the form of data processing, mathematical modeling.

The development of mathematical methods of environmental forecasting is necessary both for the optimization of environmental management and for the serious scientific substantiation of programs in the field of environmental quality management and nature protection. Mathematical models of aquatic ecosystems allow us to describe non-equilibrium dynamic processes in hydrobiocenoses under various external influences, such as changes in water temperature, surface illumination, water or biogenic load, meteorological conditions. The building of such models requires a large amount of information about the parameters of geophysical, geochemical, biological, and other natural processes, the comprehension of which is impossible without the involvement of modern information technologies [7, 8].

Materials and methods of the study. The use of the tools for mathematical modeling of surface water quality is explained by the need to assess the quality of surface water.

Dynamic models of biological systems adapted to aquatic plant microorganism communities were systematized and analyzed. Such qualitative analytical models of the dynamics of aquatic ecosystems describe the mechanisms of functioning of ecosystems: changes in the state of the aquatic environment due to physicochemical transformations, the dynamics of the living component of the ecosystem (biological community).

The body. Outcomes. When studying aquatic ecosystems, problems arise in the complexity and diversity of the processes studied. These difficulties are compounded by the inaccuracy of available data and the lack of information at all. Nevertheless, modeling under such conditions is possible and even necessary. Without models, the quantitative characteristics of processes cannot be assessed more or less satisfactorily. And the fact is that the data obtained while different field surveys are often not well consistent with time, space, measurement methods, etc. For answers to questions about the functioning of a large aquatic ecosystem, one can not do without mathematical modeling [9].

Multiple coherent models, rather than one, can be used to improve the modeling of aquatic ecosystems. Start with some idealized, closed models, then move to open ones, possibly more appropriate to real objects. Let us describe one implementation of such an approach [10].

First, we build so-called *closed* ecosystem models. Closedness is understood as a closed state of a model, quantity. It relates only to the modeled characteristics and is conditional in nature, in contrast to the closedness or isolation of the ecosystem itself [11].

Having considered the properties and modified the closed models, it is necessary to move to quantitatively open models. The solution of open models is calculated using the properties of closed model solutions, such a transition is possible. The open model becomes appropriate. However, it is necessary to build a set of models interconnected for the selected ecological system. In short, they must be coherent.

Let the state of the ecosystem at time t be described by an n-dimensional vector $x(t)$, the components of which are the essence of the biomass or mass of the isolated blocks of the ecosystem. We build the model on the basis of a self-sustained system of differential equations of the approach, formula (1) [10, p. 1401]:

$$\frac{\partial x}{\partial t} = f(x,a) \tag{1}$$

Parameters of vector a, the content of which is unknown or inaccurately determined.

Let us write another model describing the same ecological system, expression (2):

$$\frac{\partial y}{\partial t} = g(y,b) \tag{2}$$

where y – is a vector of biomass, b – unknown parameters.

The studied object is the same and the vectors x, y have the following relationship:

$$h(x,y,c) = 0 \tag{3}$$

c – unknown or inaccurately known set of parameters [12].

Shall the parameters, structures, and solution be coherent, it can be determined that the models are coherent. In the works of Doctor of Physics and Mathematics, Professor A.I. Abakumov, these concepts are formalized: first, he calculated the complete derivative $\frac{\partial h}{\partial t}$ across the solutions of the systems of equations (1) and (2) and found the parameters a, b, c from the condition of the proximity of this derivative to zero, expression (4) [12, p.1403]:

$$\left\| \frac{\partial h(x,y,c)}{\partial x} \cdot f(x,a) + \frac{\partial h(x,y,c)}{\partial y} g(y,b) \right\| \xrightarrow[(a,b,c)]{} \inf \tag{4}$$

Where $\|\cdot\|$ denotes the norm in the space of measurable square-integrated functions. Condition (4) means coherence of velocities, phase variables, not the *x, y* variables themselves, as recorded in condition (3). At the same time, we make the models coherent not only in solutions of ordinary differential equations but also in some set of changes in the variables *x, y*. So, formally, the *coherence* of the models is determined by condition (4) to select the parameters *a, b, c*. However, the choice of a set of variable parameters is informal, to a large extent, it is determined by the content characteristics of the models [12, p.16].

Discussion. Within the framework of a multi-model approach to the study of natural biosystems, closed and open models, models with and without regard to the internal state of organisms, are considered. In closed models there is a continuum set of positive equilibrium solutions, in open models there is a finite set of isolated non-negative equilibrium solutions. In models without taking into account the intracellular content of the substance, it is possible to prove the stability of equilibrium solutions, including using the signs of structural stability (sign-stability). In models with regard to intracellular nutrient content, this can be done with some limitations, although on the basis of computational experiments there remains an idea of the correctness of the stability properties for this group of models as a whole.

Signs of stability of aquatic ecosystems in mathematical models have been identified: the properties of sustainability of solutions are model signs of stability of aquatic ecosystems.

Conclusion. In conclusion, it should be noted that in order to study aquatic ecosystems and communities of water organisms in detail, it is necessary to use a set of mathematical models for the same object. This determines the effectiveness of using the information at our disposal. Data on aquatic ecosystems are collected from a variety of sources and all of these data can be collected and applied in a set of coherent models.

The stability properties of solutions are model signs of stability of aquatic ecosystems.

REFERENCES

1 Kostitsyn V.A. Evolution of the Atmosphere, Biosphere and Climate / trans. from French; ed. from ed. N.N. Moisseyeva. - M.: Nauka, 1984. - 96 p.

2 Jorgensen S.E., Loffler H., Rast W., Straskraba M. Lake and Reservoir Management // Elsevier. – 2005. – 512 p.

3 Menshutkin V.V. Mathematical Modeling of Populations and Communities of Aquatic Animals. - L.: Nauka, 1971. - 196 p.

4 Rational Use of Water Resources of the Azov Sea Basin/ ed. I.I. Vorovich. - M.: Nauka, 1981. - 360 p.

5 Poluektov R.A., Pykh Yu.A., Shvytov I.A. Dynamic Models of Ecological Systems. - L.: Gidrometeoizdat, 1980. - 288 p.

6 Shitikov V.K., Rosenberg G.S., Zinchenko T.D. Quantitative Hydroecology: Methods of Systemic Identification. - Togliatti: IEVB RAS, 2003. - 463 p.

7 Antonovskiy M.Ya., Semyonov S.M. Mathematical Methods of Environmental Forecasting. - M.: Znaniye, 1978. - 67 p.

8 Ismailova A.A. Theoretical Foundations and Concept of Solutions to Problems of Studying Biological Objects and Environmental Monitoring by Methods of Mathematics and Computer Science // Theory of Models and Algebra: Mater. International Conf. – Karaganda, 2012. – P. 79-81.

9 Ismailova A., Abakumov A., Adamov. A. Modeling of Microbial Communities of Plant Organisms in Aquatic Ecosystem //Internationl Journal of INFORMATION. – 2014. – Vol.17, No.1. – P. 209-218.

10 Abakumov a.i., Giricheva E.E. Multi-Model Approach to the Study of Aquatic Ecosystems // Izvestiya Samara Scientific Center of the RAS. – 2009. – Vol. 11, No. 1(7). – P. 1399-1402.

11 Gubanov V.G., Degermenji A.G. On the Confinement of the Biotic Cycle of Matter in Supraorganismal Systems (Methodological Aspect). Biotic Confinement Measure// Essays of Ecological Biophysics. Novosibirsk: Publishing House of Siberian Branch of RAS, 2003. – P. 318-333.

12 Abakumov A.I., Pak S.Ya. Dynamics of Phytoplankton Abundance Depending on Mineral Nutrition (Mathematical Models) // Informatics and Control Systems. – 2010. – No. 3 (25). – P. 10-19.

Р. Н. МОЛДАШЕВА[1], А. А. ИСМАИЛОВА[1], А. М. ЗАДАҒАЛИ[2]

[1]С. Сейфуллин атындағы Қазақ агротехникалық университеті,
Нұр-Сұлтан, Қазақстан
[2]Л. Н. Гумилев атындағы Еуразия ұлттық университеті, Нұр-Сұлтан, Қазақстан

МАТЕМАТИКАЛЫҚ МОДЕЛЬДЕРДЕ СУ ЭКОЖҮЙЕЛЕРІНІҢ ТҰРАҚТЫЛЫҒЫ БЕЛГІЛЕРІ

Қазіргі уақытта су ресурстары туралы түрлі деректер жинақталған, дегенмен су экожүйелерінің ағымдағы жағдайын бағалау үшін қолжетімді параметрлер болып гидробиологиялық және гидрохимиялық көрсеткіштер болып табылады, олар суқоймалары ішіндегі экологиялық жағдайды бағалау үшін негіз бола алады. Еліміздің көлдері мен өзендері туралы көп жылдық және әр түрлі деректерді жүйелеу, ақпараттық-коммуникациялық технологияларды пайдаланбай, су экожүйесінің жағдайын бағалау және болжау үшін математикалық аппараттарды қолдану мүмкін емес. Су экожүйелерін сапалы математикалық модельдеу және су экожүйелерін зерттеудің ақпараттық-аналитикалық жүйесін дамыту су қоймасы мен оның экожүйесі туралы әртүрлі сапалы мәліметтер базасын, деректерді басқару және өңдеу құралдарын, сондай-ақ су экожүйесінің жұмыс атқаруының математикалық модельдерінің кешенін қамтитын өзекті мәселе болып табылады. Су экожүйелерін сапалы математикалық модельдеу және су экожүйелерін зерттеудің ақпараттық-аналитикалық жүйесін дамыту су қоймасы мен оның экожүйесі туралы әртүрлі сапалы мәліметтер базасын, деректерді басқару және өңдеу құралдарын, сондай-ақ су экожүйесінің жұмыс істеуінің математикалық модельдерінің кешенін қамтитын өзекті мәселе болып табылады.

Зерттеулер ақпараттық технологияларға, деректерді статистикалық өңдеуге және математикалық модельдеуге негізделген. Математикалық модельдер дифференциалдық теңдеулер жүйесіне негізделген, шешімдері жеке есептеу бағдарламалары мен компьютерлік пакеттерінің (Maple, Matlab, Mathematics және т.б.) көмегімен шешіледі.

Мүмкіндігінше, модельдерде шешімдер қасиеттеріне аналитикалық зерттеулер жүргізіледі, ең алдымен бұл стационарлық немесе кеңістіктегі біртекті шешімдерге, сондай-ақ шешімдердің асимптотикалық қасиеттеріне қатысты. Су қоймаларының экожүйесінің төменгі трофикалық деңгейлері зерттеледі, себебі бұл су экожүйелерінің жұмысын анықтайды. Фитопланктонның түрлік құрамы су қоймасының экологиялық жағдайының индикаторы болып табылады.

Фитопланктонның сандық сипаттамалары негізінде су экожүйесінің биологиялық өнімділігі есептеледі. Судың физика-химиялық сипаттамалары су қоймалардың ластануы мен фитопланктон үшін минералды тамақтану құрамы туралы қорытынды жасауға мүмкіндік береді.

Түйін сөздер: *математикалық модельдеу, ақпараттық-аналитикалық жүйе, су экожүйесі, деректер қоры.*

Р. Н. МОЛДАШЕВА[1], А. А. ИСМАИЛОВА[1], А. М. ЗАДАҒАЛИ[2]

*[1]Казахский агротехнический университет им. С. Сейфуллина,
Нур-Султан, Казахстан*
*[2]Евразийский национальный университет им. Л. Н. Гумилева,
Нур-Султан, Казахстан*

ПРИЗНАКИ СТАБИЛЬНОСТИ ВОДНЫХ ЭКОСИСТЕМ В МАТЕМАТИЧЕСКИХ МОДЕЛЯХ

На сегодняшний день накоплены различные данные о водных ресурсах, однако доступными параметрами для оценки текущего состояния водных экосистем остаются гидробиологические и гидрохимические показатели, которые могут быть основой для оценки экологической ситуации внутри водоема. Систематизация многолетних и разнообразных данных об озерах и реках страны, применение математических аппаратов для оценки и прогнозирования состоянии водной экосистемы невозможно без использования информационно-коммуникационных технологий. Качественное математическое моделирование водных экосистем и разработка Информационно-аналитическа система исследования водных экосистем является актуальной задачей, включающей базы разнокачественных данных о водоеме и его экосистеме, средства управления данными и их обработки, а также комплекс математических моделей функционирования экосистемы водоема.

В основу исследований положены информационные технологии, статистическая обработка данных и математическое моделирование. Математические модели основаны на системах дифференциальных уравнений, решения ищутся с помощью собственных вычислительных программ и вычислительных компьютерных пакетов (Maple, Matlab, Mathematics и т.д.). По возможности в моделях проводятся аналитические исследования свойств решений, в первую очередь это касается стационарных или однородных по пространству решений, а также асимптотических свойств решений. Исследуются нижние трофические уровни экосистемы водоема, так как этим определяется функционирование водных экосистем. Видовой состав фитопланктона является индикатором экологического состояния водоема. На основе количественных характеристик фитопланктона рассчитывается биопродуктивность водной экосистемы. Физико-химические характеристики воды позволяют сделать выводы о загрязнении водоема и составе минерального питания для фитопланктона.

Ключевые слова: *математическое моделирование, информационно-аналитическая система, водная экосистема, база данных, фитопланктон.*