

**E. S. VITULYOVA¹*, K. N. KADYRZHAN², A. B. KADYRZHAN²,
D. B. SHALTYKOVA³, I. E. SULEIMENOV³**

¹*al-Farabi Kazakh National University, Almaty, Kazakhstan;*

²*Almaty University of Power Engineering and Telecommunications named after Gumarbek Daukeev, Almaty, Kazakhstan;*

³*National Engineering Academy of the Republic of Kazakhstan, Almaty, Kazakhstan.*

*e-mail: *Lizavita@list.ru*

DIAGNOSTICS OF SUBSURFACE OBJECTS USING UNMANNED AERIAL VEHICLES: USING GENERALIZED FOURIER OPTICS METHODS

It is shown that there is a variety of subsurface objects, the diagnosis of which can be reduced to establishing the shape of the interfaces between media with different refractive indices. An example of this kind of objects are man-made structures created to conduct painful actions, in particular, dugouts, underground tunnel systems, etc. It is shown that the surfaces inherent in such objects can be represented through equivalent circuits built on the basis of a set of paraboloids. It is proven that the characteristics of each of these paraboloids can be determined empirically through the use of analogues of optical systems operating in the radio range. These schemes are aimed at empirically determining the focal length of individual paraboloids included in the equivalent surface scheme by scanning the tunable focal length of the diagnostic system (in the simplest case, a diagnostic lens). It is shown that the application of this approach is closely related to the establishment of singular points of the wave front (singular points of the caustic), which are identified by methods of generalized Fourier optics using the stationary phase method. It is shown that with this approach, the identification of paraboloids that make up the equivalent optical scheme of the surface under study becomes unambiguous.

Key words: *subsurface objects, radio holography, generalized Fourier optics, focal length, scanning, equivalent circuit, unmanned aerial vehicles.*

**E. С. ВИТУЛЁВА¹*, Қ. Н. ҚАДЫРЖАН², А. Б. ҚАДЫРЖАН²,
Д. Б. ШАЛТЫКОВА³, И. Э. СУЛЕЙМЕНОВ³**

¹*әл-Фараби атындағы Қазақ ұлттық университеті, Алматы, Қазақстан;*

²*Ғұмарбек Даукеев атындағы Алматы энергетика және байланыс университеті,
Алматы, Қазақстан;*

³*Қазақстан Республикасының Ұлттық инженерлік академиясы, Алматы, Қазақстан.*

*e-mail: *Lizavita@list.ru*

ҰШҚЫШСЫЗ ҰШАТЫН АППАРАТТАРДЫҢ КӨМЕГІМЕН БЕТКІ ҚАБАТ АСТЫНДАҒЫ ОБЪЕКТИЛЕРДІ ДИАГНОСТИКАЛАУ: ЖАЛПЫЛАМА ФУРЬЕ ОПТИКАЛЫҚ ӘДІСТЕРІН ҚОЛДАНУ

Диагнозды әртүрлі сыну көрсеткіштері бар орталар арасындағы интерфейстердің пішінін орнатуға дейін азайтуға болатын жер қойнауының әртүрлі объектілері бар екендігі көрсетілген. Мұндай объектілердің мысалы ретінде ұрыс қимылдары үшін жасалған жасанды құрылыстар, атап айтқанда, блиндаждар, жерасты туннельдік жүйелері және т.б. Мұндай объектілерге тән беттерді параболоидтар жиынтығы негізінде салынған эквивалентті схемалар арқылы көрсетуге

болатыны көрсетілген. Осы параболоидтардың әрқайсысының сипаттамаларын радио диапазонында жұмыс істейтін оптикалық жүйелердің аналогтарын қолдану арқылы эмпирикалық түрде анықтауға болатыны дәлелденді. Бұл сұлбалар диагностикалық жүйенің (ең қарапайым жағдайда диагностикалық линза) реттелетін фокустық арақашықтықты сканерлеу арқылы эквивалентті беттік схемаға енгізілген жеке параболоидтардың фокустық аралығын эмпирикалық түрде анықтауға бағытталған. Бұл тәсілді қолдану стационарлық фазалық әдісті қолдану арқылы жалпыланған Фурье оптикасының әдістерімен анықталатын толқындық фронттың ерекше нүктелерін (каустиканың ерекше нүктелерін) орнатумен тығыз байланысты екендігі көрсетілген. Бұл тәсілмен зерттелетін беттің эквивалентті оптикалық сұлбасын құрайтын параболоидтарды анықтау бір мағыналы болатыны көрсетілген.

Түйін сөздер: беткі қабат астындағы объектілер, радиоголография, жалпылама Фурье оптикасы, фокустық қашықтық, сканерлеу, эквиваленттік схема, ұшқышсыз ұшу аппараттары.

**Е. С. ВИТУЛЁВА^{1*}, К. Н. КАДЫРЖАН², А. Б. КАДЫРЖАН²,
Д. Б. ШАЛТЫКОВА³, И. Э. СУЛЕЙМЕНОВ³**

¹Казахский национальный университет имени аль-Фараби, Алматы, Казахстан;

²Алматинский университет энергетики и связи имени Гумарбека Даукеева,
Алматы, Казахстан;

³Национальная инженерная академия Республики Казахстан, Алматы, Казахстан.
e-mail: *Lizavita@list.ru

ДИАГНОСТИКА ПОДПОВЕРХНОСТНЫХ ОБЪЕКТОВ ПРИ ПОМОЩИ БЕСПИЛОТНЫХ ЛЕТАТЕЛЬНЫХ АППАРАТОВ: ИСПОЛЬЗОВАНИЕ МЕТОДОВ ОБОБЩЕННОЙ ФУРЬЕ-ОПТИКИ

Показано, что существует разновидность подповерхностных объектов, диагностика которых может быть сведена к установлению формы границ раздела сред, обладающих различными коэффициентами преломления. Примером такого рода объектов являются рукотворные сооружения, создаваемые для ведения боевых действий, в частности, блиндажи, системы подземных тоннелей и т.д. Показано, что поверхности, присущие такому роду объектам, могут быть представлены через эквивалентные схемы, построенные на основе совокупности параболоидов. Доказывается, что характеристики каждого из таких параболоидов могут быть определены эмпирически за счет использования аналогов оптических систем, функционирующих в радиодиапазоне. Данные схемы ориентированы на эмпирическое определение фокусного расстояния отдельных параболоидов, входящих в состав эквивалентной схемы поверхности, за счет сканирования перестраиваемого фокусного расстояния диагностирующей системы (в простейшем случае – диагностирующей линзы). Показано, что применение данного подхода тесным образом связано с установлением особых точек волнового фронта (особых точек каустики), которые выявляются методами обобщенной Фурье-оптики с использованием метода стационарной фазы. Показано, что при таком подходе выявление параболоидов, составляющих эквивалентную оптическую схему исследуемой поверхности, становится однозначным.

Ключевые слова: подповерхностные объекты, радиоголография, обобщенная Фурье-оптика, фокусное расстояние, сканирование, эквивалентная схема, беспилотные летательные аппараты.

Currently, radio holography methods are being actively developed [1-3], aimed, among other things, at diagnosing subsurface objects [4,5].

Subsurface objects can have different structures, including objects with pronounced boundaries that occupy an important place. This partly relates to ore bodies, but the nature

of the fighting on the territory of Ukraine, as well as the Gaza Strip, shows that remote diagnostics of man-made subsurface objects (underground premises of Azovstal, converted into defensive structures in Mariupol, tunnels built by the Hamas movement in the Gaza Strip) is also of interest Gas, etc.). Such objects obviously have pronounced boundaries, which, in relation to the propagation of radio waves, become phase boundaries at which classical phenomena of refraction of electromagnetic radiation occur. There is no doubt that this fact, among other things, actualizes the creation of groups of UAVs operating as a systemic whole, for which there are general methodological prerequisites [6].

In [7], it was shown that it is possible to synthesize an analogue of optical systems through the use of groups of unmanned aerial vehicles (UAVs). In this case, each of the UAVs performs an operation to introduce a phase shift into the field distribution created when the object under study is irradiated with radio waves (Fig. 1). Taken together, this approach makes it possible to synthesize, for example, an analogue of a lens or other optical system intended for diagnosing a subsurface object. Theoretical justification for the possibility of using a set of groups of discrete wavefront converters instead of elements such as a lens was given in [8], where it was shown that the classical Huygens-Fresnel principle can be reduced to a discrete form. Accordingly, any radiation converter operating in reflection can be represented by the circuit in Fig. 2.

The discrete form of this principle is of particular interest for the radio range. Indeed, from the materials of the cited work it follows that the wave field (provided that inhomogeneous waves characterized by a complex value of the wave vector can be excluded from consideration) can be reconstructed on the basis of amplitudes recorded at discrete points located half the length apart waves. In the case when large-sized subsurface objects are studied, it is advisable to switch to the use of relatively long waves (small-scale inhomogeneities with a characteristic size of the order of several tens of centimeters are most often of no interest). Accordingly, analogues of the lenses mentioned above can be assembled based on groups of UAVs spaced from each other at a distance of several meters or more.

The possibility of synthesizing large-sized lenses based on groups of UAVs, in turn, creates definite advantages for diagnosing subsurface objects with pronounced boundaries.

Proving this statement and demonstrating the possibility of its practical implementation is the goal of this work.

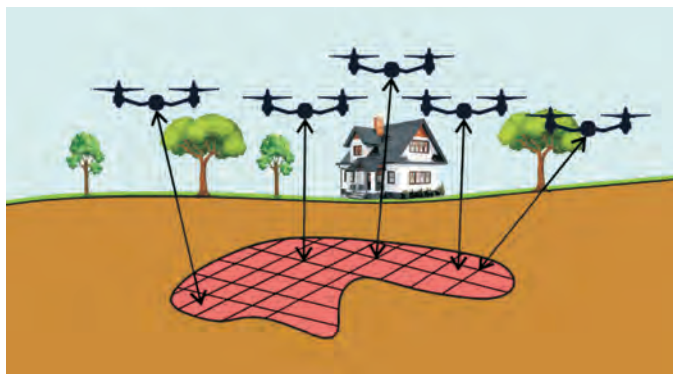


Fig. 1 – Carrying out diagnostics of a subsurface object using a group of UAVs.

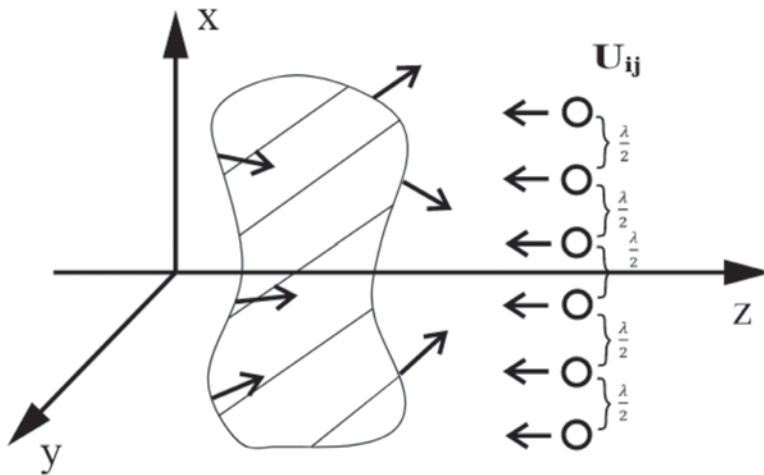


Fig.2 – Circuit of a wave converter operating in reflected radiation (an example of a body with pronounced boundaries).

Starting from Fig. 2, let's consider the situations when radio holographic reconnaissance is used to study objects that have clear boundaries. Examples in this regard are underground structures, ore bodies, oil-bearing layers, etc., as noted above.

The conversion of radio frequency radiation in this case occurs at the interfaces of media with different refractive indices. In the general case, the interaction of a wave oscillation with the interfaces between media is described by the Fresnel formulas, which, among other things, show that as a result of such interaction appearance of reflected waves.

Consequently, from the point of view of radio holography tasks, homogeneous objects of complex shape can be considered as a combination of two reflective surfaces of complex shape.

Reflection from a system containing two mirrors, generally speaking, leads to interference effects associated, among other things, with the phenomenon of multipath interference [9].

If, however, the reflection coefficient remains low, which, as follows from Fresnel's formulas, is a typical situation, then it is permissible to consider reflective surfaces, which can be conditionally divided into "left" and "right" separately.

Consequently, the problem under consideration comes down to the question of studying the geometry of a certain reflecting surface and finding its basic characteristics.

There is, however, an important caveat. The interfaces between media encountered in practice are, as a rule, close to fractal. More precisely, they have inhomogeneities of various scales, and the "smallest" inhomogeneities are often not of interest. This nuance is significant from the point of view of choosing the length of radio waves used for diagnostics.

For a given specific wavelength λ , any inhomogeneities (deviations of the reflecting surface from the plane) can be classified as follows

- large: the characteristic scale of heterogeneity significantly exceeds λ ;
- average: the characteristic scale is comparable to λ ;
- small: the characteristic scale of heterogeneity is significantly less than λ .

Inhomogeneities of the latter type are excluded from consideration due to the choice of wavelength. Under this condition, the wave does not interact with this kind of inhomogeneity.

The choice of wavelength is therefore dictated by the particular scale of inhomogeneities that must be identified and characterized. As will be clear from what follows, the most convenient choice is one in which the characteristic scale of the inhomogeneities of interest significantly exceeds the wavelength. In many ways, this case corresponds to the approximation of geometric optics.

Generalized Fourier optics methods [10,11] make it possible to move from a reflective surface of complex shape to its equivalent scheme shown in Fig. 3; the surface is approximated by a set of paraboloids, which intuitively seems quite natural.



Fig. 3 – Equivalent scheme of a reflective surface of complex shape.

There is, however, an important caveat. The shape of the surface in question is unknown. Approximating paraboloids must be found based on measurement data, which forces one to turn to the use of generalized Fourier optics methods to describe reflection from a surface of complex shape. This description is as follows [10,11].

To the surface S described by the formula,

$$z = z(x, y) \tag{1}$$

a plane wave falls, which, in relation to further considerations, can be conveniently represented as

$$f = \exp\left(ik\left[\alpha x + \beta y - \sqrt{1 - \alpha^2 - \beta^2} z\right]\right) \tag{2}$$

The minus sign for the square root in expression (2) was chosen in accordance with Fig. 2. It is assumed that the incident wave propagates from right to left, and the reflected wave propagates from left to right.

On the plane S , given by relation (1), this wave creates the following field distribution

$$u_{in} = \exp\left(ik\left[\alpha_0 x + \beta_0 y - \sqrt{1 - \alpha_0^2 - \beta_0^2} z(x, y)\right]\right) \tag{3}$$

The difference between notation (2) and formula (3) is that in formula (2) z is a variable, i.e. can take arbitrary values, and in formula (43) z is a function of two other coordinates. Otherwise, function (2) depends on three variables and describes the field distribution in the entire space, and function (3) depends only on two variables and specifies the field distribution on a specific surface. This is what justifies the use of the u_{in} notation for this function.

Reflection within the framework of a scalar description (with polarization effects excluded from consideration) is described through a change in phase by π .

Accordingly, the distribution of the “output” field on the surface under consideration is given by the expression

$$u_{out} = -\exp\left(ik\left[\alpha x + \beta y - \sqrt{1 - \alpha^2 - \beta^2} z(x, y)\right]\right) \tag{4}$$

The difference between formula (4) and formula (3) is that distribution (4) is formed by waves belonging to another branch of the spatial frequency spectrum, i.e. waves propagating in the opposite general direction.

Mathematically, this is expressed in the fact that the spectrum of spatial frequencies of the field distribution u_{out} can be determined through the integral [10,11]

$$A_{out} = -\int \exp(ik\bar{\xi} - \bar{r}) \exp(-ik\bar{\xi} + \bar{r}) dS \tag{5}$$

Where $\bar{\xi} - \bar{r} = \alpha_0 x + \beta_0 y - \sqrt{1 - \alpha_0^2 - \beta_0^2} z(x, y)$;

$$\bar{\xi} + \bar{r} = \alpha x + \beta y + \sqrt{1 - \alpha^2 - \beta^2} z(x, y)$$

For clarity, let us consider the case of the paraxial approximation, when approximately we can set $\sqrt{1 - \alpha^2 - \beta^2} \approx 1$, which is true at small angles. Then

$$A_{out} = -\int \exp(ik[(\alpha_0 - \alpha)x + (\beta_0 - \beta)y - 2z(x, y)]) dx dy \tag{6}$$

It can be seen that this is the Fourier transform of the function of the optical thickness of the mirror, which has the form

$$T_A = -\exp(-ik2z(x, y)) \tag{7}$$

Integral (4) allows the following interpretation. The field distribution corresponding to an individual spectral component is first multiplied by function (7), and then the Fourier transform is calculated, which corresponds to finding the spectrum of spatial frequencies in the approximation of classical (paraxial) Fourier optics.

Let us consider the behavior of integral (5), assuming that the inequality holds

$$a \gg \lambda \tag{8}$$

where a is the characteristic scale of changes in the relief of the reflecting surface.

Then we apply the stationary phase method to integral (5), which consists of the following.

Provided that the parameter k is large, which in the case under consideration physically corresponds to the fulfillment of condition (8), in the integral of the form

$$J = -\int \exp(ikf(x, y)) dx dy \tag{9}$$

the integrand can be replaced by a Taylor series expansion up to quadratic terms in the vicinity of the stationary point. The coordinates of this point (x_0, y_0) are determined from the condition

$$\begin{cases} \frac{\partial f}{\partial x}(x_0, y_0) = 0 \\ \frac{\partial f}{\partial y}(x_0, y_0) = 0 \end{cases} \tag{10}$$

Accordingly, the integral goes into

$$J = -\int \exp(ik[f(x_0, y_0) + a_{11}x^2 + 2a_{12}xy + a_{22}y^2])dxdy \tag{11}$$

Where $a_{11} = \frac{\partial^2 f}{\partial x^2} \Big|_{x_0, y_0}$; $a_{12} = \frac{\partial^2 f}{\partial x \partial y} \Big|_{x_0, y_0}$; $a_{22} = \frac{\partial^2 f}{\partial y^2} \Big|_{x_0, y_0}$.

Integral (11) is calculated explicitly. Physically, the meaning of the transition to expansion in the integrand is as follows. Everywhere, except in the vicinity of the stationary point, the integrand is rapidly oscillating. Accordingly, the contribution from the neighborhood of all points except the stationary point can be neglected, since rapidly oscillating vibrations cancel each other out.

Consequently, the stationary phase method allows the calculation of integrals of the type under consideration to solve algebraic equations.

This condition also meets the expression for the distribution of the field of the wave reflected from the surface under consideration. In accordance with the methodology described above, this distribution can be obtained by passing from the spatial frequency spectrum to the field amplitude at point \vec{r}_0 as

$$u_{out}(\vec{r}_0) = -\iint \exp(ik\vec{\xi} - \vec{r}) \exp(-ik\vec{\xi} + \vec{r})dS \exp(ik\vec{\xi} + \vec{r})d\Sigma \tag{12}$$

The stationary phase method is also applicable to this integral, but with the difference that in this case the integration is carried out over four variables.

To move to an equivalent circuit of a reflective surface of complex shape, the most essential, however, is the consideration of degenerate stationary points, which, as shown in [12, 13], correspond to a caustic surface.

As applied to the problem under consideration, the physical meaning of degenerate stationarity points is as follows. Each local segment of a surface other than a plane can be considered as a parabolic/spherical mirror. When a plane wave falls on it, it focuses at a certain point. The set of all such points constitutes a caustic surface. Mathematically, these are degenerate stationary points for which the determinant of the matrix of second partial derivatives vanishes. In this case, one cannot limit oneself to the expansion of the phase function in a Taylor series accurate to quadratic terms, and it is also necessary to take into account terms of higher orders.

Note, however, that the degeneration of stationarity points can be deeper, i.e. the caustic surface [13] has its own singular points.

Physically, such points correspond to the operation of a focusing mirror in the paraxial optics approximation.

Indeed, let us consider the reflection of radiation from a spherical mirror. If we consider each of its local sections as an independent focusing element, then the caustic surface shown in this figure will be formed.

However, among all such points there is a special one. A spherical mirror achieves approximately the same focusing as a parabolic one, which can be demonstrated, among other things, by an elementary construction using geometric optics.

Such a special point corresponds to the focus of a spherical mirror; it is equal to half its radius of curvature.

Consequently, the approach aimed at identifying degenerate points of stationarity, developed within the framework of generalized Fourier optics, actually makes it possible to divide a reflecting surface of complex shape into a finite set of mirrors that are close to parabolic.

In fact, this means that the surface under study is replaced by a set of areas in which it is approximated by paraboloids.

The task of radio-holographic diagnostics in this case, therefore, comes down to finding the parameters and location of each of these paraboloids, for which, among other things, you can use directed radio beams (more precisely, narrowly directed radiation), which interacts with a local section of the reflecting surface. It is significant that a group of UAVs is capable of generating highly targeted radiation of even relatively long wavelengths by ensuring phase correlation between the emitters. This approach is known to be quite widely used in practice [14].

We also note that the approximation under consideration differs significantly from the one used in computational methods. This can be seen most clearly by considering the spline method. In accordance with it, an arbitrary curve is divided into finite sections, in which it is approximated by parabolas, and the parameters of the parabolas are selected so that the curve remains continuous. In relation to the spline method, the division of the curve into sections approximated by parabolas can be arbitrary.

The method, based on finding increasingly degenerate points of stationarity, makes it possible to demonstrate that a similar approximation should be chosen based on the type of surface under study, i.e. the partition shown in Fig. 3 is not arbitrary.

Thus, there is an important class of problems in the field of radioholography that can be reduced to establishing the geometric characteristics of paraboloids that approximate the phase boundaries.

To carry out diagnostics based on this principle, in turn, it is important to provide adequate analysis/classification of caustic points inherent in reflected wave fronts.

This approach, in the future, creates the prerequisites for diagnosing subsurface objects, which does not require solving problems of mathematical physics designed to describe the interaction of radio frequency radiation with objects of complex shape. The practical implementation of this approach can be based on the use of groups of UAVs.

Acknowledgements. This research has been funded by the Science Committee of the Ministry of Science and Higher Education of the Republic of Kazakhstan (Grant No. AP15473224).

REFERENCES

- 1 Zhang, X., Kang, H., Zuo, Y., Lou, Z., Wang, Y., & Qian, Y. (2019). Near-Field Radio Holography of Slant-Axis Terahertz Antennas. *IEEE Transactions on Terahertz Science and Technology*, 10(2), 141-149.
- 2 Xu L., Zuo Y. Design study on the digital correlator using for radio holography //International Conference on Intelligent and Interactive Systems and Applications. – Springer, Cham, 2016. – C. 377-382.
- 3 Igarashi, K., Pavelyev, A., Hocke, K., Pavelyev, D., Kucherjavenkov, I. A., Matyugov, S., ... & Yakovlev, O. (2000). Radio holographic principle for observing natural processes in the atmosphere and retrieving meteorological parameters from radio occultation data. *Earth, planets and space*, 52, 893-899.
- 4 Ivashov, S. I., Razevig, V. V., Vasiliev, I. A., Zhuravlev, A. V., Bechtel, T. D., & Capineri, L. (2011). Holographic subsurface radar of RASCAN type: Development and applications. *IEEE Journal of Selected Topics in Applied Earth Observations and Remote Sensing*, 4(4), 763-778.
- 5 Cherepenin, V. A., Zhuravlev, A. V., Chizh, M. A., Kokoshkin, A. V., Korotkov, V. A., Korotkov, K. V., & Novichikhin, E. P. (2017). Reconstruction of subsurface radio holograms fully and partially measured by different methods. *Journal of Communications Technology and Electronics*, 62, 780-787.
- 6 Suleimenov, I. E., Gabrielyan, O. A., Malenko, S. A., Vitulyova, Y. S., & Nekita, A. G. (2021). Algorithmic Basis Of Battle Neural Networks And Crisis Phenomena In Modern Society. In D. Y. Krapchunov, S. A. Malenko, V. O. Shipulin, E. F. Zhukova, A. G. Nekita, & O. A. Fikhtner (Eds.), *Perishable And Eternal: Mythologies and Social Technologies of Digital Civilization*, vol 120. *European Proceedings of Social and Behavioural Sciences* (pp. 247-255).
- 7 Bayana B. Ermukhambetova, Grigoriy A. Mun, Sherniyaz B. Kabdushev, Aruzhan Bulatovna Kadyrzhan, Kaisarali K. Kadyrzhan, Yelizaveta S. Vitulyova, Ibragim E. Suleimenov New approaches to the development of information security systems for unmanned vehicles *Indonesian Journal of Electrical Engineering and Computer Science* Vol. 31, No. 2, August 2023, pp. 810~819
- 8 Vitulyova Y., Matrassulova D., Suleimenov I., Bakirov A. “Discrete form of the Huygens-Fresnel principle: to the multi-dimensional analog of the Nyquist–Shannon sampling theorem”, *International Journal of Information Technology*, accepted 13.08.23.
- 9 Suleimenov, I. E., & Kuranov, A. L. (1997). Multibeam interference in systems with ideal translational invariance. *Optics and Spectroscopy*, 82(3), 445-450.
- 10 Suleimenov, I. E., & Tolmachev, Y. A. (1994). Generalized Fourier optics. I. Reflection of monochromatic radiation from mirrors of arbitrary shape. *Optics and spectroscopy*, 77(1).
- 11 Suleimenov, I. E., & Tolmachev, Y. A. (1994). Generalized Fourier optics: II. application of the stationary phase method to the description of wavefront propagation and reflection. *Optics and spectroscopy*, 77(3), 422-428.
- 12 Suleimenov, I. E., & Tolmachev, Y. A. (1995). Generalized fourier optics: III. Description of the wave front reflection from nonplanar mirrors in terms of local curvature. *Optics and Spectroscopy*, 78(1).
- 13 Suleimenov, I. E., & Tolmachev, Y. A. (1995). Classification of singularities of caustic surfaces. *Optics and Spectroscopy*, 79(1), 156-158.
- 14 A. Ripak, V. Khaikin and M. Lebedev, “Aperture Field Recovery of a Reflector Radio Telescope using Phase Shifting Holography,” 2020 7th All-Russian Microwave Conference (RMC), Moscow, Russia, 2020, pp. 162-166, doi: 10.1109/RMC50626.2020.9312237.