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GPR SOUNDING SOURCE CALIBRATION

The paper describes the process of calibrating a GPR sounding source, provides a mathematical basis for this process, and presents the results of calibration based on data measured in a sand quarry, where the electromagnetic parameters of the probed medium are known. The obtained parameter values characterizing the behavior of the source can now be considered known and used to numerically solve inverse problems to determine unknown electromagnetic parameters of the medium using algorithms that are not built into the GPR software.
Key words: *GPR, source calibration, inverse problem.*

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GPR ДЫБЫС КӨЗІН КАЛИБРЛЕУ

Жұмыста GPR зондтау көзін калибрлеу процесі сипатталған, осы процестің математикалық негізі берілген және зондталатын ортаның электромагниттік параметрлері белгілі құм карьерінде өлшенген деректер негізінде калибрлеу нәтижелері берілген. Көздің мінез-құлқын сипаттайтын алынған параметр мәндерін енді белгілі деп санауға болады және GPR бағдарламалық жасақтамасына салынбаған алгоритмдерді пайдалана отырып, ортаның белгісіз электромагниттік параметрлерін анықтау үшін кері есептерді сандық түрде шешу үшін пайдалануға болады.
Түйін сөздер: *GPR, бастапқы калибрлеу, кері мәселе.*

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КАЛИБРОВКА ИСТОЧНИКА ЗОНДИРОВАНИЯ ГЕОРАДАРА

В работе описан процесс калибровки источника зондирования георадара, приведено математическое обоснование этого процесса, приведены результаты калибровки по данным, измеренным на песочном карьере, где электромагнитные параметры зондируемой среды являются известными. Полученные значения параметров, характеризующие поведение источника, теперь могут считаться известными и использоваться для численного решения обратных задач по определению неизвестных электромагнитных параметров среды с использованием алгоритмов, которые не являются встроенными в математическое обеспечение георадара.

Ключевые слова: *георадар, калибровка источника, обратная задача.*

Introduction. The source was calibrated on a medium whose electromagnetic parameters were known. The work shows the possibility of determining the sounding source of GPR.

Mathematically, the problem is equivalent to solving the inverse problem of determining the right side of a differential equation.

The built-in GPR software typically uses interpretation algorithms that use the travel times of electromagnetic waves, so the exact form of the function describing the source as a function of time is not required [1,2]. If there is a wish to use other algorithms for determining the electromagnetic properties of a medium (for example, [3]), then to use them one need to know the source function with good accuracy. In addition, during operation, the GPR may get physical damage, which may lead to changes in the function of the source.

The paper proposes to determine the source by solving a direct problem in the frequency domain. The main difficulty in determining the source function is that the GPR probes the medium at several fixed time frequencies, so the image of the source function is known only for a narrow range of time frequencies. What makes solving the problem easier is that the behavior of the source function is known, and, therefore, in order to establish its form, one need to determine only a few parameters on which it depends. The solution to the differential equation in the frequency domain is written out in analytical form, and the inverse problem of determining the source is reduced to finding the minimum of a functional.

1. Mathematical basis for calibration (refinement of source shape). Let's consider a model of a medium where the electromagnetic parameters of the medium depend only on the depth z: $\{z<0\}$ is air, $\{z>0\}$ is medium.

Let the electromagnetic field be excited by a source of external current of the following type:

$$j(r,\phi,t)=(j_r, j_\phi j_z)^T=(0, j_\phi, 0)^T \ , \ j_\phi(r,\phi,t)=f(t)g(r)\delta(z-z_*) \qquad (1)$$

where z_* is the source coordinate on the axis Oz, $z_*<0$ (the source locates in the air) and the value z_* is quite small, and the function $g(r)=\theta(r_0-r)$, where $r_0>0$ is the source parameter, $\theta(x)$ – Heaviside θ – function.

The electromagnetic field is described by Maxwell's equations. The medium is characterized by electromagnetic parameters of the medium: permittivity $\epsilon_0\epsilon$ (ϵ_0 is permittivity of vacuum, $\epsilon \geq 1$ is relative permittivity of medium), conductivity σ and magnetic permeability μ. The permittivity of vacuum $\epsilon_0 = 8{,}854 \cdot 10^{-12}$ (F/m), for most geophysical media $\mu = 4\pi \cdot 10^{-7}$ (H/m). In air $\epsilon = 1$ and $\sigma = 0$ ($z < 0$), in medium $\epsilon = \epsilon_m$ and $\sigma = \sigma_m$ ($z > 0$), and ϵ_m and σ_m are known constants.

Since the medium is isotropic and the source does not depend on the angle φ, the Maxwell equations can be written in a cylindrical coordinates, and the components of the electromagnetic field will not depend on the angle φ. Taking into account the type of source (1), of the six components of the electromagnetic field, three will be non-zero: E_φ, H_r and H_z (see, for example, [4]). For the component E_φ the following differential equation can be obtained:

$$\mu \epsilon_0 \epsilon E_{\phi,tt} + \mu\sigma E_{\phi,t} = E_{\phi,zz} + ((1/r)(rE_\phi)_r)_r - \mu j_{\phi,t} \tag{2}$$

An equation for which the initial conditions, boundary conditions and gluing conditions at the air-medium boundary hold:

$$E_\phi\Big|_{t<0} \equiv 0, \quad E_\phi\Big|_{r<0} \equiv 0, \quad [E_\phi]_0 = 0, \quad [E_{\phi,z}]_0 = 0, \tag{3}$$

Let measurements be made on the surface $z = 0$:

$$E_\phi\Big|_{z=+0} = \varphi(r,t), \tag{4}$$

Our goal is to determine the GPR source function (see (1)), assuming that the electromagnetic component E_φ satisfies the direct problem (2)-(3), and the additional boundary condition (4) is known. This problem will be solved in the frequency domain.

We use the Laplace and Hankel transformations:

$$u(v,z,p) = \int_0^\infty e^{-pt} \int_0^\infty rE_\varphi(r,z,t) J_1(vr)\,dr\,dt, \tag{5}$$

where $p = \alpha + i2\pi f$ is the Laplace transform parameter, α is an attenuation parameter, f is a time frequency (Hz), $J_1(r)$ is the Bessel function of 1st order.

We will use the following notation: $\varphi(v, p)$ - image for function, $\varphi(r, t)$, $f(p)$ – Laplace image for the function $f(t)$, $g(v)$ – the Hankel image for the function $g(r)$. It is easy to see [5, p. 697, # 6.561.1] that

$$g(v) = \int_0^\infty r\,\theta(r_0 - r) J_1(vr)\,dr = \frac{\sqrt{\pi}}{2}\frac{r_0}{v}\,(J_1(r_0 v)\mathcal{H}_0(r_0 v) - J_0(r_0 v)\mathcal{H}_1(r_0 v)),$$

where $J_n(r)$ and $\mathcal{H}_n(r)$ are Bessel and Struve functions of n-th order (see, for example, [5,6].

The function $u(v, z, p)$ satisfies the following equation:

$$u_{zz} - (v^2 + p^2\mu\epsilon_0\epsilon + p\mu\sigma)u = \mu p f(p) g(v) \sigma(z - z_*),\ z \in (-\infty, \infty). \tag{6}$$

Using solutions to equation (6) on the intervals $(-\infty, z_*)$, $(z_*, 0)$, and $(0, \infty)$, assuming decay of the solution to equation (6) at infinity, using gluing conditions at the air-medium interface point and at point z_*, going to the limit $z_* \to 0$, the following relation can be obtained:

$$-\mu p f(p) g(v) = (k_m + k_0) u(v,+0,p) = (k_m + k_0)\varphi(r,t)\ , \tag{7}$$

Here $k_m = \sqrt{v^2 + p^2\mu\epsilon_0\epsilon_m + p\mu\sigma_m}$, $\quad k_0 = \sqrt{v^2 + p^2\mu\epsilon_0}$, $\quad \mathrm{Re}\{r_m\} > 0, \quad \mathrm{Re}\{r_0\} > 0.$

Using the relation (7), we can propose an algorithm for determining the source function $f(t)$. This will be described in Section 4.

2. Measurements on a known medium. Measurements were taken in a sand quarry near the village of Sabyndy, Akmola region. For measurements, we used the GPR "Loza-B", whose operating frequencies are $f^{(1)} = 150$ and $f^{(2)} = 300$ (MHz).

Fig. 1 (a) shows a medium model.

a) b)

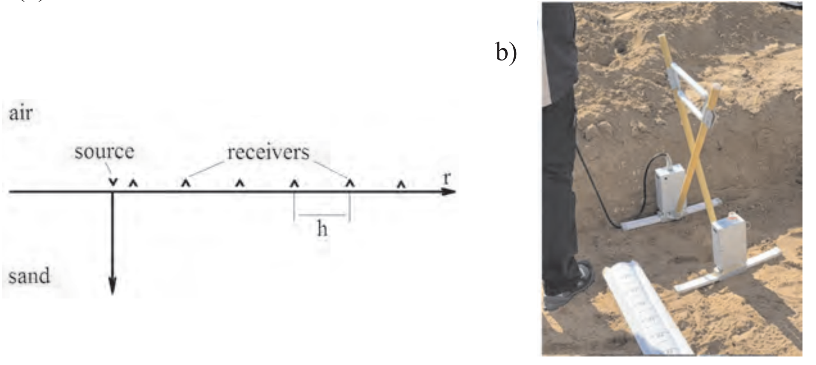

Figure 1 – (a) – Medium model and placement of the source and receivers,
(b) - Taking measurements using the GPR "Loza-B" in a sand quarry.

The known medium was dry sand, the electromagnetic properties of which were characterized by relative dielectric permeability $\epsilon_m = 6$ and conductivity $\sigma_m = 0{,}0005$ (S/m).

Electromagnetic waves were excited by a source of the form (1), electromagnetic wave receivers were located on the surface $z = 0$ with a step $h = 0{,}5$ (m) and were located on the same straight line (See Fig. 1 (a)).

Fig. 1 (b) shows the measurement process using GPR.

3. Data preparation. As noted above, the receivers recorded the electromagnetic field component $E\varphi$ at points $(r_i, 0, t),\quad i = 1, N_r,\quad t \in [0, T],\quad T = 50$ (ns). Receivers are located at points r_i (see Fig. 1 (a)), the distance between them is h. The maximum distance of the receiver L_r from the source of electromagnetic waves is at least $5z_s$, since when electromagnetic waves propagate deep into the medium, their amplitude decreases, the thickness of the skin layer is given by the formula $z_s = 1/\sqrt{\pi f \mu \sigma}$, and, consequently, the amplitude electromagnetic waves passing through the medium at this distance decreases by more than 140 times, i.e. the approximation $E_\varphi(r, 0, t) = 0$ for $r \geq L_r$ is quite appropriate.

Data $E_\varphi(r_i, 0, t)$ were filtered (see, for example, [7,8]).

From Kotelnikov's theorem [9] it follows that the Fourier transform frequencies used should not exceed the value of the Nyquist frequency $v_{max} = 1/2h$ [10]. Since the Bessel function $J_1(r)$ behaves like a damped sinusoid, the Nyquist frequency can be used to determine the maximum spatial frequency v. From [3] it follows that v must satisfy the inequality $v \leq 2\pi f \sqrt{\mu \epsilon_m}$. Therefore, we use these two inequalities to select the frequency v for computing (5). Therefore,

$$u(v, z, p) = \int_0^\infty e^{-pt} \int_0^\infty r E_\varphi(r, z, t) J_1(vr)\,dr\,dt$$
$$\approx \sum_{J=0}^{N_T} e^{-pt_j} \sum_{i=1}^{N_r} r_i E_\phi(r_i, z, t_j) J_1(vr_i)$$

can be calculated. The attenuation parameter α is selected from the condition $\alpha = 2z_s\sqrt{\mu_0\epsilon_m}$ [3], time frequencies f are selected from the intervals $[f^{(k)} - \delta, f^{(k)} + \delta]\,(k = 1,2)$, and τ is the time step.

4. Numerical method for determining source parameters. GPR has the ability to generate sounding signals only on several time frequencies, in our case on two. This means that for each specific sounding frequency $f^{(k)}$ are used in the transformation (5) only temporary frequencies from very narrow intervals $[f^{(k)} - \delta, f^{(k)} + \delta]\,(k = 1,2)$.

When designing an antenna that generates a probing electromagnetic signal, the manufacturer informs the GPR user about the type of source, i.e. the form of the function $f(t)$ is known. It depends on the design of the GPR and, as a rule, this function can be well approximated by a decreasing sinusoid: $\mathrm{R}e^{-2\pi f_0 t}\sin(2\pi f_0 t)$, where f_0 is the dominant frequency (in GPR it coincides with one of the operating frequencies) and R is the source amplitude.

We use the unknown GPR source function $f(t)$ in the form $f(t) = \rho \mathrm{R}e^{-2\pi\beta f_0 t}\sin(2\pi\gamma f_0 t)$ and its Laplace image is $f(p) = \dfrac{2R\pi\rho\gamma f_0}{(p - 2\pi\beta f_0)^2 + (2\pi\gamma f_0)^2}$.

It is obvious that each of the parameters ρ, β, and γ are corrective and affects the exact form of the function $f(t)$, i.e. the parameters ρ, β, and γ are responsible for calibrating the GPR source.

Using the equality (7), we define the cost function Φ(ρ, β, γ) as follows:

$$\Phi(\rho,\beta,\gamma) = \sum\nolimits_{n=1}^{N}\left|\phi(v, p_n) + \mu g(v)\frac{p_n}{k_{m.n} + k_{0,n}}\frac{2R\pi\rho\gamma f_n}{(p - 2\pi\beta f_n)^2 + (2\pi\gamma f_n)^2}\right|^2$$

here $k_{m,n} = \sqrt{v^2 + p_n^2\mu_0\epsilon_0\epsilon_m + p_n\mu\sigma_m}$, $k_{0,n} = \sqrt{v^2 + p_n^2\mu_0\epsilon_0}$, $p_n = \alpha + i2\pi f_n$, $(n = \overline{1, N})$, frequencies f_n are selected from the interval $[f^{(k)} - \delta, f^{(k)} + \delta]$ (usually with equal steps).

Thus, the problem of determining the unknown GPR source function $f(t)$ is reduced to minimizing the cost function (8) by determining the unknown parameters ρ, β, and γ.

The minimum of the cost function (8) can be found using the conjugate gradient method (see, for example, [11]). Gradient $\nabla\Phi(\rho,\beta,\gamma) = (\Phi'_\rho, \Phi'_\beta, \Phi'_\gamma)$ where

$$\Phi'_\rho = \begin{cases} 0, & \text{if } \begin{array}{l}\Phi(\rho + \delta_{\rho,}\beta,\gamma) \le \Phi(\rho,\beta,\gamma),\\ \Phi(\rho - \delta_{\rho,}\beta,\gamma) \le \Phi(\rho,\beta,\gamma),\end{array} \\ \dfrac{\Phi(\rho + \delta_{\rho,}\beta,\gamma) - \Phi(\rho - \delta_{\rho,}\beta,\gamma)}{2\delta\rho}, & \text{else,} \end{cases}$$

components Φ'_β, Φ'_γ are calculated using similar formulas. It should be noted that formulas of the form (9) for calculating the gradient were first used in the work [12] and further confirmed their effectiveness in many works (see, for example, [3, 13-19]. Explanations of why they are built this way can be found here [20]. The step δ_ρ (and steps for calculating Φ'_β and Φ'_γ) is sought when solving the inverse problem on simulated data. The value δ_ρ is selected for which the solution to the problem of minimizing the cost function (8) is found in the least number of iterations of the minimization process. Using our experience and experience of [3, 13-19], it can be selected as $\delta_\rho = 10^{-6}$.

To start solving the problem of minimizing the function (8) by searching for unknown parameters ρ, β, and γ using the conjugate gradient method, it is necessary to specify an initial approximation. These can be the following values: $\rho_{[0]} = 1$, $\beta_{[0]} = 1$, and $\gamma_{[0]} = 1$, or the values obtained during the last calibration of the source.

Remark: It depends on GPR construction, sometimes other the pulse forms can be used to model the source for the function $f(t)$: Gaussian, Ricker source (“mexican hat”), Puzyrev source and etc. In this case, the proposed mathematical algorithm will not change in any way; the change will only affect the representation for the Laplace image $f(p)$.

5. Calibration. The source is calibrated for each of the operating frequencies of GPR. Operating frequency $f^{(k)}$, operating frequency interval from which frequencies f were selected to write the function (8), their number N is given in Tab. 1. The parameters ρ, β and γ determined after practical measurements (see Fig. 1 (b)) are collected in Tab. 2

Table 1.

k	$f^{(k)}$, (MHz)	Interval (MHz)	N
1	150	[130,170]	5
2	300	[280,320]	5

Table 2.

k	$f^{(k)}$, (MHz)	ρ	β	γ
1	150	1.01	1.13	1.21
2	300	1.03	1.09	1.18

6. Testing. We use the found parameters ρ, β and γ (see Tab. 2). Let us determine the relative permittivity and conductivity of clayey sandy soil near the sand quarry.

The GPR data are measured and processed, setting $\alpha = 0$. From the relation (7) it follows $k_m = -k_0 - 2i\pi\mu f \dfrac{f(2i\pi f)g(v)}{\phi(v, 2i\pi f)}$, and in this case $\epsilon_m = \dfrac{v^2 - \mathrm{Re}\{k_m^2\}}{\mu\,\epsilon_0 (2\pi f)^2}$, $\sigma_m = \dfrac{\mathrm{Im}\{k_m^2\}}{\mu 2\pi f}$.

The test result is shown in Tab. 3:

Table 3

k	$f^{(k)}$, (MHz)	permittivity of soil	restored permittivity	conductivity of soil	restored conductivity
1	150	5.00	5.25	0.0010	0.0012
2	300	5.00	5.14	0.0010	0.0013

A satisfactory result was obtained for reconstructing the electromagnetic parameters of the soil.

Conclusion. The paper presents the process of calibrating the GPR source function, provides a mathematical basis for this process, and presents the results of calibration based on data measured in a sand quarry, where the electromagnetic parameters of the probed medium are known. The obtained parameter values characterizing the behavior of the source can now be considered known and used to numerically solve inverse problems to determine unknown electromagnetic parameters of the medium using algorithms that are not built into the GPR software.

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